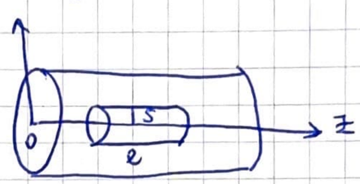
DATE GIADA SELVA S4763440

HW SET 1

ADVANCED ELECTRODYNAMICS

(1) Gauss law: $\oint \vec{E} \cdot d\vec{a} = \frac{Q_{ENCL}}{\varepsilon_0}$

i) $s < a$ we take a Gaussian surface inside the inner cylinder, coaxial to it



$Q_{ENCL} = \int_V \rho \, d\tau = \rho \int_V d\tau = \rho \cdot \pi s^2 \ell$

since E is ∥ to the basis of the cylinder and $|\vec{E}|$ is constant on the lateral surface:

$\oint \vec{E} \cdot d\vec{a} = \int_{SL} E \, da = E \, 2\pi s \cdot \ell$

Gauss's law:

$$E \cdot 2\pi s \ell = \rho \cdot \frac{\pi s^2 \ell}{\varepsilon_0} \Rightarrow \vec{E} = \frac{\rho s}{2\varepsilon_0} \hat{s}$$

ii) $a < s < b$

Gaussian surface between the cylinders:

$Q_{ENCL} = \int_V \rho \, d\tau = \rho \int_V d\tau = \rho \cdot \pi a^2 \ell$

$\oint \vec{E} \cdot d\vec{a} = E \cdot 2\pi s \ell$

Gauss's law

$$E \cdot 2\pi s \ell = \rho \frac{\pi a^2 \ell}{\varepsilon_0} \Rightarrow \vec{E} = \frac{\rho a^2}{2\varepsilon_0 s} \hat{s}$$

iii) $s > b$

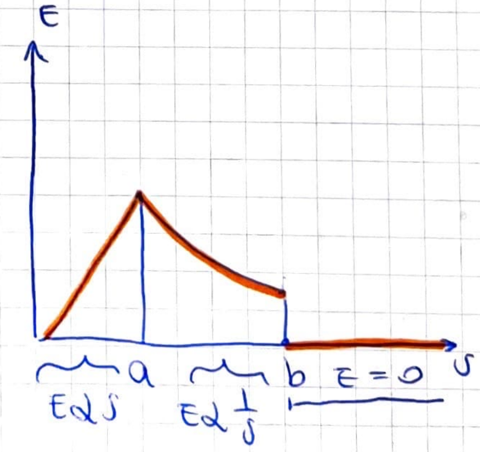
Gaussian surface outside the cylinders

$Q_{ENCL} = 0$

$\oint \vec{E} \cdot d\vec{a} = E \cdot 2\pi s \ell$

Gauss law

$E \cdot 2\pi s \ell = 0 \Rightarrow E = 0$



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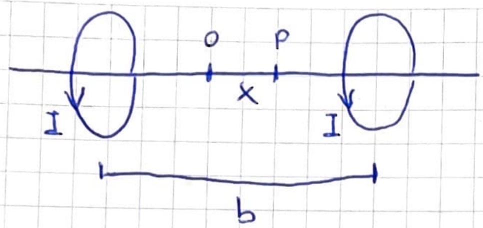
(2)



a) MAGNETIC FIELD IN P

it is the sum of the magnetic fields of the two circular loops:

$$B = \mu_0 \frac{I a^2}{2}\left[\frac{1}{[a^2+(\frac{b}{2}+x)^2]^{\frac{3}{2}}} + \frac{1}{[a^2+(\frac{b}{2}-x)^2]^{\frac{3}{2}}}\right]$$

b) $$\frac{\partial B}{\partial x} = \mu_0 \frac{I a^2}{2}\left[\frac{-\frac{3}{2}\cdot 2(\frac{b}{2}+x)}{[a^2+(\frac{b}{2}+x)^2]^{\frac{5}{2}}} + \frac{+\frac{3}{2}\cdot 2(\frac{b}{2}-x)}{[a^2+(\frac{b}{2}-x)^2]^{\frac{5}{2}}}\right]$$

$$= 3\mu_0 \frac{I a^2}{2}\left[\frac{-(\frac{b}{2}+x)}{[a^2+(\frac{b}{2}+x)^2]^{\frac{5}{2}}} + \frac{(\frac{b}{2}-x)}{[a^2+(\frac{b}{2}-x)^2]^{\frac{5}{2}}}\right]$$

$$\frac{\partial^2 B}{\partial x^2} = 3\mu_0 I \frac{a^2}{2}\left[-\left(\frac{b}{2}+x\right)\frac{(-\frac{5}{2})\cdot 2(\frac{b}{2}+x)}{[a^2+(\frac{b}{2}+x)^2]^{\frac{7}{2}}} - \frac{1}{[a^2+(\frac{b}{2}+x)^2]^{\frac{5}{2}}} - \right.$$

$$\left. - \left(\frac{b}{2}-x\right)\cdot\frac{(-\frac{5}{2})2(\frac{b}{2}-x)}{[a^2+(\frac{b}{2}-x)^2]^{\frac{7}{2}}} - \frac{1}{[a^2+(\frac{b}{2}-x)^2]^{\frac{5}{2}}}\right]$$

c) $$\left.\frac{\partial^2 B}{\partial x^2}\right|_{x=0} = 3\mu_0 I \frac{a^2}{2}\left[\frac{+\frac{b}{2}\cdot 5\cdot\frac{b}{2}}{[a^2+(\frac{b}{2})^2]^{\frac{7}{2}}} - \frac{1}{[a^2+(\frac{b}{2})^2]^{\frac{5}{2}}} + \right.$$

$$\left. + \frac{\frac{b}{2}\cdot 5\cdot\frac{b}{2}}{[a^2+(\frac{b}{2})^2]^{\frac{7}{2}}} - \frac{1}{[a^2+(\frac{b}{2})^2]^{\frac{5}{2}}}\right] =$$

$$= 3\mu_0 I \frac{a^2}{2}\left[-\frac{2}{[a^2+(\frac{b}{2})^2]^{\frac{5}{2}}} + 2\cdot\frac{5(\frac{b}{2})^2}{[a^2+(\frac{b}{2})^2]^{\frac{7}{2}}}\right]$$

$$= \frac{3\mu_0 I a^2}{[a^2+(\frac{b}{2})^2]^{\frac{7}{2}}}\left[-\left[a^2+\left(\frac{b}{2}\right)^2\right]+5\left(\frac{b}{2}\right)^2\right] =$$

$$= \frac{3\mu_0 I a^2}{[a^2+(\frac{b}{2})^2]^{\frac{7}{2}}}\left(b^2-a^2\right)$$

TO MAKE $\frac{\partial^2 B}{\partial x^2} = 0$

WE MUST HAVE $b = a$

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in the middle point O the first derivative $\frac{\partial B}{\partial x}$ is zero and if we put the coils at a distance equal to the radius, b = a, also the second derivative

$\left.\frac{\partial^2 B}{\partial x^2}\right|_{x=0}$ is zero.

It means that in this way we can obtain a near uniform magnetic field.

d) B at x = 0 when a = b

$$B = \mu_0 I \frac{a^2}{2}\left[\frac{2}{\left[a^2+\left(\frac{a}{2}\right)^2\right]^{\frac{3}{2}}}\right] = \mu_0 I a^2 \left[\frac{1}{\left(\frac{5}{4}a^2\right)^{\frac{3}{2}}}\right]$$

$$= \mu_0 I a^2 \cdot \frac{1}{5^{\frac{3}{2}} \cdot \frac{1}{(2^2)^{\frac{3}{2}}} (a^2)^{\frac{3}{2}}} = \frac{\mu_0 I a^2 2^3}{5^{\frac{3}{2}} \cdot a^3}$$

$$B(0) = \frac{8 \mu_0 I}{5^{\frac{3}{2}} a}$$

(3) a) $\rho = \frac{Q}{V} = e \cdot N_A \cdot \frac{d}{M}$ CHARGE DENSITY

since $e \cdot N_A$ is the charge of 1 mole

d is the density, $d = \frac{M}{V}$, M MOLE MASS

$$\rho = 1.6 \cdot 10^{-19} C \cdot 6 \cdot 10^{23} \text{mol} \cdot \frac{9 \text{ gr/cm}^3}{64 \text{ gr/mol}} = 1.4 \times 10^4 C / cm^3$$

b) $J = \frac{I}{A}$ A = CROSS SECTION

$J = nev = \rho v$

$$\Rightarrow \frac{I}{\pi r^2} = \rho v \rightarrow v = \frac{I}{\pi r^2 \rho} = \frac{1A}{\pi\left(\cdot\frac{1}{2}\cdot 10^{-1}\right)^2 cm^2 \cdot 1.4 \times 10^4 C/cm^3}$$

$v = 9.1 \times 10^{-3}$ cm/s v_{DRIFT} of the electrons

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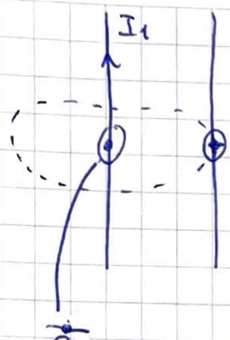
c)



$\vec{B}$ Due to I_1 : $B_1 = \dfrac{\mu_0 I_1}{2\pi d}$ ⊗

$\vec{B}$ Due to I_2 : $B_2 = \dfrac{\mu_0 I_2}{2\pi d}$ ⊙

FORCE ON WIRE 2

$\vec{F} = I_2 \int d\vec{\ell} \times \vec{B_1}$

$F = I_2 \int d\ell \, \dfrac{\mu_0 I_1}{2\pi d} = \mu_0 \dfrac{I_1 I_2}{2\pi d} \int d\ell$

FORCE PER UNIT LENGHT :

$F_M = \mu_0 \dfrac{I_1 I_2}{2\pi d} = \dfrac{4\pi \cdot 10^{-7} \cdot 1 \cdot 1}{2\pi \cdot 10^{-2}} = 2 \times 10^{-5}\ N$

d) $E = \dfrac{1}{2\pi \varepsilon_0} \dfrac{\lambda}{d}$ electric field at distance d from an infinite wire

λ = linear charge density

$F_E = \dfrac{1}{2\pi \varepsilon_0} \dfrac{\lambda_1 \lambda_2}{d}$ $\qquad I = \lambda v$

$F_E = \dfrac{1}{2\pi \varepsilon_0} \dfrac{I_1 I_2}{v^2 d} = \dfrac{1}{2\pi \cdot 8.9 \times 10^{-12}} \cdot \dfrac{1}{(9.1 \times 10^{-3})^2 \cdot 10^{-2}} = 2.2 \times 10^{18}\ N$

electric force is so much greater

$\dfrac{F_E}{F_M} = \dfrac{1}{2\pi \varepsilon_0} \dfrac{I_1 I_2}{v^2 d} \cdot \dfrac{2\pi d}{\mu_0 I_1 I_2} = \dfrac{1}{v^2 \varepsilon_0 \mu_0} = \dfrac{c^2}{v^2}$

since $c^2 = \dfrac{1}{\varepsilon_0 \mu_0}$