DATE

HW 2 ADVANCED ELECRODYNAMICS GIADA SELVA

① $\varepsilon_0 \approx 8.85 \cdot 10^{-12}$ F/m F = FARAD

$$\frac{F}{m} = \frac{C}{V} \frac{1}{m} = \frac{A \cdot S}{Kg \cdot m^2 \cdot s^{-3} A^{-1} m} = \frac{A^2 s^4}{Kg\, m^3}$$

$$\varepsilon_0 \approx 8.85 \cdot 10^{-12} A^2 s^4 Kg^{-1} m^{-3}$$

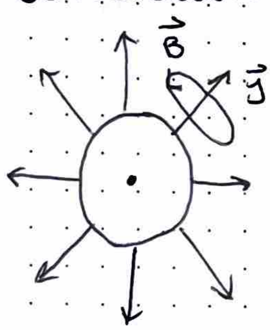
$\mu_0 \approx 1.26 \cdot 10^{-6}$ H/m H = HENRY

$$\frac{H}{M} = \frac{Kg \cdot m^2}{s^2 A^2} \cdot \frac{1}{m} = Kg\, m\, s^{-2} A^{-2} \qquad \mu_0 \approx 1.26 \cdot 10^{-6} Kg\, m\, s^{-2} A^{-2}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = \frac{1}{\sqrt{8.85 \cdot 10^{-12} \times 1.26 \cdot 10^{-6} \underbrace{A^2 s^4 Kg^{-1} m^{-3} Kg\, m\, s^{-2} A^{-2}}_{s^2 m^{-2}}}}$$

$c = 3 \times 10^8$ m/s

② In this situation we have a spherically symmetric distribution of current;



if we take an amperian loop, there is current enclosed in it and we expect to find a magnetic field $\vec{B}$.
But we are in a symmetric situation and if we choose another loop we will find a different circulation of $\vec{B}$

It is not possible to have a symmetry for $\vec{B}$ that is only out or inwards, because magnetic monopoles do not exists

The only conclusion is that $\vec{B}$ is zero. The explanation is in the fact that $\vec{B}$ depends not only on the current but also on the rate of change of the electric field $\frac{d\vec{E}}{dt}$. Thus 2 contributions cancel out and $\vec{B} = 0$

DATE

(3) The solutions of the inhomogeneous wave equations are the retarded potentials:

$$V(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}',t_r)}{\mathscr{r}}\,d\tau'$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}',t_r)}{\mathscr{r}}\,d\tau'$$ where $t_r = t - \frac{\mathscr{r}}{c}$ is the retarded time

we want to prove that they satisfy the lorentz gauge conditions:

$$\nabla\cdot\vec{A} = -\mu_0\varepsilon_0\frac{\partial V}{\partial t}$$

PROOF:

$$\nabla\cdot\vec{A} = \frac{\mu_0}{4\pi}\int \nabla\cdot\left(\frac{\vec{J}}{\mathscr{r}}\right)d\tau'$$

$$\nabla\cdot\left(\frac{\vec{J}}{\mathscr{r}}\right) = \frac{1}{\mathscr{r}}(\nabla\cdot\vec{J}) + \vec{J}\cdot\left(\nabla\cdot\frac{1}{\mathscr{r}}\right) \quad (1)$$ PRODUCT RULE

$$\nabla'\cdot\left(\frac{\vec{J}}{\mathscr{r}}\right) = \frac{1}{\mathscr{r}}(\nabla'\cdot\vec{J}) + \vec{J}\cdot\left(\nabla'\frac{1}{\mathscr{r}}\right) \quad (2)$$ where ∇' is $\frac{\partial}{\partial r'}$

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}' \quad , \quad \mathscr{r} = |\vec{r} - \vec{r}'|$$

$$\nabla\left(\frac{1}{\mathscr{r}}\right) = -\nabla'\left(\frac{1}{\mathscr{r}}\right)$$

$$\nabla\cdot\left(\frac{\vec{J}}{\mathscr{r}}\right) = \frac{1}{\mathscr{r}}(\nabla\cdot\vec{J}) - \vec{J}\cdot\left(\nabla'\frac{1}{\mathscr{r}}\right) =$$ APPLYING (2)

$$= \frac{1}{\mathscr{r}}(\nabla\cdot\vec{J}) + \frac{1}{\mathscr{r}}(\nabla'\cdot\vec{J}) - \nabla'\cdot\left(\frac{\vec{J}}{\mathscr{r}}\right)$$

$$\vec{J} = \vec{J}(\vec{r}',t_r) \quad , \quad t_r = t - \frac{\mathscr{r}}{c} \quad , \quad \vec{\mathscr{r}} = \vec{r} - \vec{r}'$$

SINCE $$\frac{df(x,y)}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$$

$$\nabla\cdot\vec{J} = \sum_i \frac{\partial J_i}{\partial x_i}(x_i', t_r) = \sum_i\left(\frac{\partial J_i}{\partial x_i'}\cdot\frac{\partial x_i'}{\partial x_i} + \frac{\partial J_i}{\partial t_r}\frac{\partial t_r}{\partial x_i}\right)$$

$$= \sum_i \frac{\partial J_i}{\partial t_r}\cdot\frac{\partial t_r}{\partial x_i}$$ SINCE $\frac{\partial x_i'}{\partial x_i} = 0$ $\quad x_i'$ DOES NOT DEPEND ON x_i

$$= -\frac{1}{c}\frac{\partial \vec{J}}{\partial t_r}\cdot\nabla\mathscr{r}$$ SINCE $t_r = t - \frac{\mathscr{r}}{c}$

$$\nabla'\cdot\vec{J} = \sum_i\left(\frac{\partial J_i}{\partial x_j'}\cdot\frac{\partial x_j'}{\partial x_i'} + \frac{\partial J_i}{\partial t_r}\cdot\frac{\partial t_r}{\partial x_i'}\right)$$

$$\sum_i \frac{\partial J_i}{\partial x_j'}\cdot\frac{\partial x_j'}{\partial x_i'} = \sum_i \frac{\partial J_i}{\partial x_i'} = \nabla'\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$ FOR THE CONTINUITY EQUATION

$$\nabla'\cdot\vec{J} = -\frac{\partial\rho}{\partial t} - \frac{1}{c}\frac{\partial\vec{J}}{\partial t_r}\cdot\nabla'\mathscr{r}$$

So:

$$\nabla\cdot\left(\frac{\vec{J}}{\imath}\right)=\frac{1}{\imath}\underbrace{\left[-\frac{1}{c}\frac{\partial\vec{J}}{\partial t_r}\cdot\nabla\imath\right]}_{\nabla\cdot\vec{J}}+\frac{1}{\imath}\underbrace{\left[-\frac{\partial\rho}{\partial t}-\frac{1}{c}\frac{\partial\vec{J}}{\partial t_r}\cdot\nabla'\imath\right]}_{\nabla'\cdot\vec{J}}-\nabla'\cdot\left(\frac{\vec{J}}{\imath}\right)$$

$$\nabla\cdot\left(\frac{\vec{J}}{\imath}\right)=-\frac{1}{\imath}\frac{\partial\rho}{\partial t}-\nabla'\cdot\left(\frac{\vec{J}}{\imath}\right)$$ SINCE $\nabla(|r-r'|)=-\nabla'(|r-r'|)$
$\nabla\imath=-\nabla'\imath$

AND

$$\nabla\cdot\vec{A}=\frac{\mu_0}{4\pi}\int\nabla\cdot\left(\frac{\vec{J}}{\imath}\right)d\tau'$$

$$\nabla\cdot\vec{A}=\frac{\mu_0}{4\pi}\left[-\frac{\partial}{\partial t}\int\frac{\rho}{\imath}\,d\tau\,d\tau'-\int\nabla'\cdot\left(\frac{\vec{J}}{\imath}\right)d\tau'\right]$$

$$\int\nabla'\cdot\left(\frac{\vec{J}}{\imath}\right)d\tau'=\oint(\vec{J}\cdot\hat{n})\,da'=0$$ DIVERGENCE THEOREM WITH $\vec{J}=0$ AT INFINITY

$$\Rightarrow\nabla\cdot\vec{A}=-\frac{\mu_0}{4\pi}\int\frac{1}{\imath}\frac{\partial\rho}{\partial t}d\tau'=-\frac{\mu_0\varepsilon_0}{4\pi\varepsilon_0}\int\frac{1}{\imath}\frac{\partial\rho}{\partial t}d\tau'=$$

$$=-\mu_0\varepsilon_0\frac{\partial}{\partial t}\left[\frac{1}{4\pi\varepsilon_0}\int\frac{\rho}{\imath}d\tau'\right]=-\mu_0\varepsilon_0\frac{\partial V}{\partial t}$$

$$\Rightarrow\boxed{\nabla\cdot\vec{A}=-\mu_0\varepsilon_0\frac{\partial V}{\partial t}}$$ That is the LORENZ GAUGE

(4) a) $\vec{J}$ is constant in time

continuity equation $\frac{\partial\rho}{\partial t}=-\nabla\cdot\vec{J}$

integrating w.r.t t:

$-\rho=-(\nabla\cdot\vec{J})t+\text{const}$

where $\rho=\rho(\vec{r},t)$, $\vec{J}=\vec{J}(\vec{r})$ and const only depends on $\vec{r}$

$\rho(\vec{r},t)=-(\nabla\cdot\vec{J})t+\text{const}$
$\rho(\vec{r},0)=\text{const}$
$\rho(\vec{r},t)=\dot{\rho}(\vec{r},0)t+\rho(\vec{r},0)$ SINCE $\frac{\partial\rho}{\partial t}=-\nabla\cdot\vec{J}$

ρ IS A LINEAR FUNCTION OF t

b) from the retarded potential it is possible to obtain the time dependent electric field:

$$(*)\ \vec{E}(\vec{r},t)=\frac{1}{4\pi\varepsilon_0}\int\left[\frac{\rho(\vec{r}',t_r)}{\imath^2}\hat{\imath}+\frac{1}{c\imath}\frac{\partial\rho(\vec{r}',t_r)}{\partial t_r}\hat{\imath}-\frac{1}{c^2\imath}\frac{\partial\vec{J}(\vec{r}',t_r)}{\partial t_r}\right]d\tau'$$

$\vec{J}$ is constant in time: $\frac{\partial\vec{J}}{\partial t_r}=0$

$$\dot{\rho}(\vec{r}',t_r)=\frac{\partial}{\partial t_r}\left(\dot{\rho}(\vec{r}',0)t_r+\rho(\vec{r}',0)\right)=\dot{\rho}(\vec{r}',0)$$

SUBSTITUTING IN (*)

$$\vec{E}(\vec{r},t)=\frac{1}{4\pi\varepsilon_0}\int\left[\frac{\dot{\rho}(\vec{r}',0)t_r+\rho(\vec{r}',0)}{\imath^2}\hat{\imath}+\frac{\dot{\rho}(\vec{r}',0)}{c\imath}\hat{\imath}\right]d\tau'$$

DATE

③ The solutions of the inhomogeneous wave equations are the retarded potentials:

$$V(\vec{r},t) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{r}\,',t_r)}{\mathscr{r}}\,d\tau'$$

$$\vec{A}(\vec{r},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{r}\,';t_r)}{\mathscr{r}}\,d\tau'$$

where $t_r = t - \frac{\mathscr{r}}{c}$ is the retarded time

we want to proove that they satisfy the lorentz gauge conditions:

$$\nabla\cdot\vec{A} = -\mu_0\varepsilon_0\frac{\partial V}{\partial t}$$

PROOF:

$$\nabla\cdot\vec{A} = \frac{\mu_0}{4\pi}\int \nabla\cdot\left(\frac{\vec{J}}{\mathscr{r}}\right)d\tau'$$

$$\nabla\cdot\left(\frac{\vec{J}}{\mathscr{r}}\right) = \frac{1}{\mathscr{r}}(\nabla\cdot\vec{J}) + \vec{J}\cdot\left(\nabla\cdot\frac{1}{\mathscr{r}}\right) \quad (1)$$ PRODUCT RULE

$$\nabla'\cdot\left(\frac{\vec{J}}{\mathscr{r}}\right) = \frac{1}{\mathscr{r}}(\nabla'\cdot\vec{J}) + \vec{J}\cdot\left(\nabla'\frac{1}{\mathscr{r}}\right) \quad (2)$$ where ∇' is $\frac{\partial}{\partial r'}$

$$\vec{\mathscr{r}} = \vec{r} - \vec{r}\,' \quad , \quad \mathscr{r} = |\vec{r} - \vec{r}\,'|$$

$$\nabla\left(\frac{1}{\mathscr{r}}\right) = -\nabla'\left(\frac{1}{\mathscr{r}}\right)$$

$$\nabla\cdot\left(\frac{\vec{J}}{\mathscr{r}}\right) = \frac{1}{\mathscr{r}}(\nabla\cdot\vec{J}) - \vec{J}\cdot\left(\nabla'\frac{1}{\mathscr{r}}\right) =$$ APPLYING (2)

$$= \frac{1}{\mathscr{r}}(\nabla\cdot\vec{J}) + \frac{1}{\mathscr{r}}(\nabla'\cdot\vec{J}) - \nabla'\cdot\left(\frac{\vec{J}}{\mathscr{r}}\right)$$

$$\vec{J} = \vec{J}(\vec{r}\,',t_r)\ , \quad t_r = t - \frac{\mathscr{r}}{c} \quad , \quad \vec{\mathscr{r}} = \vec{r} - \vec{r}\,'$$

SINCE $\frac{df(x,y)}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt}$

$$\nabla\cdot\vec{J} = \sum_i \frac{\partial J_i}{\partial x_i}(x_i',t_r) = \sum_i\left(\frac{\partial J_i}{\partial x_i'}\cdot\frac{\partial x_i'}{\partial x_i} + \frac{\partial J_i}{\partial t_r}\frac{\partial t_r}{\partial x_i}\right)$$

$$= \sum_i \frac{\partial J_i}{\partial t_r}\cdot\frac{\partial t_r}{\partial x_i}$$ SINCE $\frac{\partial x_i'}{\partial x_i} = 0$ x_i' DOES NOT DEPEND ON x_i

$$= -\frac{1}{c}\frac{\partial\vec{J}}{\partial t_r}\cdot\nabla\mathscr{r}$$ SINCE $t_r = t - \frac{\mathscr{r}}{c}$

$$\nabla'\cdot\vec{J} = \sum_i\left(\frac{\partial J_i}{\partial x_j'}\cdot\frac{\partial x_j'}{\partial x_i} + \frac{\partial J_i}{\partial t_r}\cdot\frac{\partial t_r}{\partial x_i}\right)$$

$$\sum_i \frac{\partial J_i}{\partial x_j'}\cdot\frac{\partial x_j'}{\partial x_i} = \sum_i \frac{\partial J_i}{\partial x_i} = \nabla'\cdot\vec{J} = -\frac{\partial\rho}{\partial t}$$ FOR THE CONTINUITY EQUATION

$$\nabla'\cdot\vec{J} = -\frac{\partial\rho}{\partial t} - \frac{1}{c}\frac{\partial\vec{J}}{\partial t_r}\cdot\nabla'\mathscr{r}$$

DATE

So:

$$\nabla\cdot\left(\frac{\vec{J}}{\imath}\right)=\frac{1}{\imath}\Big[\underbrace{-\frac{1}{c}\frac{\partial\vec{J}}{\partial t_r}\cdot\nabla\imath}_{\nabla\cdot\vec{J}}\Big]+\frac{1}{\imath}\Big[\underbrace{-\frac{\partial\rho}{\partial t}-\frac{1}{c}\frac{\partial\vec{J}}{\partial t_r}\cdot\nabla'\imath}_{\nabla'\cdot\vec{J}}\Big]-\nabla'\cdot\left(\frac{\vec{J}}{\imath}\right)$$

$$\nabla\cdot\left(\frac{\vec{J}}{\imath}\right)=-\frac{1}{\imath}\frac{\partial\rho}{\partial t}-\nabla'\cdot\left(\frac{\vec{J}}{\imath}\right)$$ SINCE $\nabla(|r-r'|)=-\nabla'(|r-r'|)$
$\nabla\imath=-\nabla'\imath$

AND

$$\nabla\cdot\vec{A}=\frac{\mu_0}{4\pi}\int\nabla\cdot\left(\frac{\vec{J}}{\imath}\right)d\tau'$$

$$\nabla\cdot\vec{A}=\frac{\mu_0}{4\pi}\left[-\frac{\partial}{\partial t}\int\frac{\rho}{\imath}d\tau\, d\tau'-\int\nabla'\cdot\left(\frac{\vec{J}}{\imath}\right)d\tau'\right]$$

$\int\nabla'\cdot\left(\frac{\vec{J}}{\imath}\right)d\tau'=\oint(\vec{J}\cdot\hat{n})\,da=0$ DIVERGENCE THEOREM WITH $J=0$ AT INFINITY.

$$\Rightarrow\nabla\cdot\vec{A}=-\frac{\mu_0}{4\pi}\int\frac{1}{\imath}\frac{\partial\rho}{\partial t}d\tau'=-\frac{\mu_0\varepsilon_0}{4\pi\varepsilon_0}\int\frac{1}{\imath}\frac{\partial\rho}{\partial t}d\tau'=$$

$$=-\mu_0\varepsilon_0\frac{\partial}{\partial t}\left[\frac{1}{4\pi\varepsilon_0}\int\frac{\rho}{\imath}d\tau'\right]=-\mu_0\varepsilon_0\frac{\partial V}{\partial t}$$

$$\Rightarrow\boxed{\nabla\cdot\vec{A}=-\mu_0\varepsilon_0\frac{\partial V}{\partial t}}$$ That is the LORENZ GAUGE

(4) a) $\vec{J}$ is constant in time

continuity equation $\frac{\partial\rho}{\partial t}=-\nabla\cdot\vec{J}$

integrating w.r.t t:

$-\rho=-(\nabla\cdot\vec{J})t+\text{const}$

where $\rho=\rho(\vec{r},t)$, $\vec{J}=\vec{J}(\vec{r})$ and const only depends on $\vec{r}$

$\rho(\vec{r},t)=-(\nabla\cdot\vec{J})t+\text{const}$

$\rho(\vec{r},0)=\text{const}$

$\rho(\vec{r},t)=\dot{\rho}(\vec{r},0)t+\rho(\vec{r},0)$ SINCE $\frac{\partial\rho}{\partial t}=-\nabla\cdot\vec{J}$

ρ IS A LINEAR FUNCTION OF t

b) from the retarded potential it is possible to obtain the time dependent electric field:

$$(*)\ \vec{E}(\vec{r},t)=\frac{1}{4\pi\varepsilon_0}\int\left[\frac{\rho(\vec{r}',t_r)\hat{\imath}}{\imath^2}+\frac{1}{c\imath}\frac{\partial\rho(\vec{r}',t_r)}{\partial t_r}\hat{\imath}-\frac{1}{c^2\imath}\frac{\partial\vec{J}(\vec{r}',t_r)}{\partial t_r}\right]d\tau'$$

$\vec{J}$ is constant in time: $\frac{\partial\vec{J}}{\partial t_r}=0$

$\dot{\rho}(\vec{r}',t_r)=\frac{\partial}{\partial t_r}\left(\dot{\rho}(\vec{r}',0)t_r+\rho(\vec{r}',0)\right)=\dot{\rho}(\vec{r}',0)$

SUBSTITUTING IN (*)

$$\vec{E}(\vec{r},t)=\frac{1}{4\pi\varepsilon_0}\int\left[\frac{\dot{\rho}(\vec{r}',0)t_r+\rho(\vec{r}',0)}{\imath^2}\hat{\imath}+\frac{\dot{\rho}(\vec{r}',0)}{c\imath}\hat{\imath}\right]d\tau'$$

$t_r = t - \frac{\mathscr{r}}{c}$

$$\vec{E}(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int\left[\frac{\dot{\rho}(\vec{r}',0)t}{\mathscr{r}^2} - \frac{\dot{\rho}(\vec{r}',0)}{\mathscr{r}c} + \frac{\rho(\vec{r}',0)}{\mathscr{r}^2} + \frac{\dot{\rho}(\vec{r}',0)}{c\mathscr{r}}\right]\hat{\mathscr{r}}\, d\tau'$$

$$\vec{E}(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\dot{\rho}(\vec{r}',0)t + \rho(\vec{r}',0)}{\mathscr{r}^2}\hat{\mathscr{r}}\, d\tau'$$

$$\vec{E}(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\int \frac{\rho(\vec{r}',t)}{\mathscr{r}^2}\hat{\mathscr{r}}\, d\tau'$$ that is the COULOMB LAW AT THE TIME t

⑤ $\vec{J}(t_r) = \vec{J}(t) + (t_r - t)\dot{\vec{J}}(t)$

$\vec{J}$ changes slowly, we take a first order approximation. From the retarded potentials, the magnetic field is

$$\vec{B}(\vec{r},t) = \frac{\mu_0}{4\pi}\int\left[\frac{\vec{J}(\vec{r}',t_r)}{\mathscr{r}^2} + \frac{\dot{\vec{J}}(\vec{r}',t_r)}{c\mathscr{r}}\right]\times\hat{\mathscr{r}}\, d\tau'$$

$\dot{\vec{J}}(t_r) = \dot{\vec{J}}(t)$ SINCE CONSIDER JUST FIRST ORDER TERMS

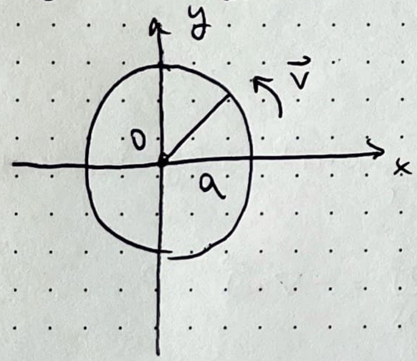
$$B(\vec{r},t) = \frac{\mu_0}{4\pi}\int\left[\frac{\vec{J}(t) + (t_r - t)\dot{\vec{J}}(t)}{\mathscr{r}^2} + \frac{\dot{\vec{J}}(t)}{c\mathscr{r}}\right]\times\hat{\mathscr{r}}\, d\tau'$$

$t_r = t - \frac{\mathscr{r}}{c}$, $t_r - t = -\frac{\mathscr{r}}{c}$

$$\vec{B}(\vec{r},t) = \frac{\mu_0}{4\pi}\int\left[\frac{\vec{J}(t)}{\mathscr{r}^2} - \frac{\dot{\vec{J}}(t)}{\mathscr{r}c} + \frac{\dot{\vec{J}}(t)}{\mathscr{r}c}\right]\times\hat{\mathscr{r}}\, d\tau'$$

$$\vec{B}(\vec{r},t) = \frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{r}',t)\times\hat{\mathscr{r}}}{\mathscr{r}^2}\, d\tau'$$ THAT IS the BIOT-SAVART LAW AT TIME t

⑥



$v = \omega a$

The LIÉNARD-WIECHERT potentials for a moving point charge are:

$$V(\vec{r},t) = \frac{1}{4\pi\epsilon_0}\frac{qc}{(\mathscr{r}c - \vec{\mathscr{r}}\cdot\vec{v})}, \quad \vec{A}(\vec{r},t) = \frac{\vec{v}}{c^2}V(\vec{r},t)$$

$\vec{w}(t) = a\cos\omega t\,\hat{x} + a\sin\omega t\,\hat{y}$

$\vec{v}(t) = \dot{w}(t) = -a\omega\sin\omega t\,\hat{x} + a\omega\cos\omega t\,\hat{y} = a\omega[-\sin(\omega t\,\hat{x}) + \cos\omega t\,\hat{y}]$

$|\vec{\mathscr{r}}| = a \qquad \vec{\mathscr{r}} = -[a\cos\omega t_r\,\hat{x} + a\sin\omega t_r\,\hat{y}] = -\vec{w}$

$t_r = t - \frac{|\vec{r} - \vec{w}(t_r)|}{c} = t - \frac{a}{c}$ it is independent from the charge position, that is always at the same distance a from the origin

and

$\vec{\mathscr{r}}\cdot\vec{v} = (\vec{r} - \vec{w}(t_r))\cdot\vec{v} = 0$

$\mathscr{r}c = ac$

$V(0,t) = \frac{1}{4\pi\epsilon_0}\frac{qc}{a\,c - 0}$

DATE

AND the scalar potential is:

$$\boxed{V(0,t) = \frac{1}{4\pi\varepsilon_0}\frac{q}{a}}$$

The vectorial potential is:

$$\vec{A}(0,t) = \frac{\vec{v}(tr)}{c^2}V(0,t) = \frac{a\omega}{c^2}\cdot\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{a}\left[-\sin(\omega tr)\hat{x} + \cos(\omega tr)\hat{y}\right]$$

$$\boxed{\vec{A}(0,t) = \frac{q\omega}{4\pi\varepsilon_0 c^2}\left[-\sin(\omega tr)x + \cos(\omega tr)\hat{y}\right]}$$

b) $\vec{E}(\vec{r},t) = \frac{q}{4\pi\varepsilon_0}\frac{\imath}{(\vec{\imath}\cdot\vec{u})^3}\left[(c^2-v^2)\vec{u} + \vec{\imath}\times(\vec{u}\times\vec{a}_c)\right]$

where $\vec{u} = c\hat{\imath} - \vec{v}(tr)$ $\qquad a_c = \ddot{\vec{w}}(t) = \dot{v}(t) = -\omega^2\vec{w}(t)$

$\vec{u} = -c\left[\cos(\omega tr)\hat{x} + \sin(\omega tr)\hat{y}\right] - \omega a\left[-\sin(\omega tr)\hat{x} + \cos(\omega tr)\hat{y}\right]$

$= -\left[c\cos(\omega tr) - \omega a\sin(\omega tr)\right]\hat{x} - \left[c\sin(\omega tr) - \omega a\cos(\omega tr)\right)\hat{y}$ (*)

$\vec{\imath}\times(\vec{u}\times\vec{a}_c) = (\vec{\imath}\cdot\vec{a}_c)\vec{u} - (\vec{\imath}\cdot\vec{u})\vec{a}_c$

$\vec{\imath}\cdot\vec{a}_c = -\vec{w}\cdot(-\omega^2\vec{w}) = \omega^2|\vec{w}|^2 = a^2\omega^2$

$\vec{\imath}\cdot\vec{u} = a\left[c\cos^2(\omega tr) - a\omega\sin(\omega tr)\cos(\omega tr) + c\sin^2(\omega tr) +\right.$
$\left. + a\sin(\omega tr)\cos(\omega tr)\right] = ac$

$v^2 = \omega^2 a^2$

$\vec{E} = \frac{q}{4\pi\varepsilon_0}\frac{a}{a^3c^3}\left[(c^2-\omega^2a^2)\vec{u}\right]$

$\vec{E} = \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{a^2c^2}\left\{\left[(\omega^2a^2-c^2)\cos(\omega tr) + \omega ac\sin(\omega tr)\right]\hat{x} + \right.$
$\left. + \left[(\omega^2a^2-c^2)\sin(\omega tr) - \omega ac\cos(\omega tr)\right]\hat{y}\right.$

$\vec{B} = \frac{1}{c}\hat{\imath}\times\vec{E}$ $\qquad \vec{B}$ IS $\perp$ AT THE xy PLANE: $\vec{B} = B\hat{z}$

$\vec{B} = \frac{1}{c}(\hat{\imath}_x E_y - \hat{\imath}_y E_x)\hat{z} = \frac{1}{c}\frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{a^2c^2}\left\{-\cos(\omega tr)\left[(\omega^2a^2-c^2)\sin(\omega tr) +\right.\right.$
$\left. + \omega ac\cos(\omega tr)\right] + \sin(\omega tr)\left[(\omega^2a^2-c^2)\cos(\omega tr) + \omega ac\sin(\omega tr)\right]\hat{z}$

$\vec{B} = \frac{q}{4\pi\varepsilon_0}\frac{1}{a^2c^3}\left[\omega ac\cos^2(\omega tr) + \omega ac\sin^2(\omega tr)\right]\hat{z}$

$\vec{B} = \frac{q}{4\pi\varepsilon_0}\frac{1}{a^2c^3}\omega ac\,\hat{z} = \frac{q}{4\pi\varepsilon_0}\frac{\omega}{ac^2}\hat{z}$

$\vec{B}$ DOESN'T VARY IN TIME