# Practice session: Physics of Stars

Kapteyn Learning Community Sirius A
Give motivations and/or derivations for your answers.

**Calculator is allowed. Closed book. No loud chewing.**

1. **Basic Principles**

   (a) Briefly describe the following terms and, if applicable, give their units in cgs:
      1. Specific intensity $I_\nu$
      2. Average intensity $J_\nu$
      3. Monochromatic Flux $F_\nu$
      4. Eddington Flux $H_\nu$
      5. Effective Temperature
      6. Local Thermodynamic Equilibrium

   (b) Briefly describe the difference between the Saha equation and the Boltzmann equation.

   (c) Name the below equation and explain each variable.

$$\frac{n_{II}}{n_I} = \frac{1}{n_e}\left(\frac{2\pi m_e kT}{h^2}\right)^{3/2}\frac{2U_{II}}{U_I}e^{-E_{ion}/kT}$$

   (d) Use this equation to calculate the number density of ionised hydrogen at a depth in a star where $T = 17000$ K. Assume that 30 percent of the electrons are from the ionisation of hydrogen (i.e., $0.3n_e = n_{II}$) and that 90 percent of the hydrogen is ionized. Also assume $\mathrm{U}_I = 2$.

2. **Variable Stars**

   (a) Briefly describe what a variable star is and give some examples.

   (b) What field of astronomy uses variable stars to study stellar interiors? What kind of observational data is needed for this?

3. **Hydrostatics**

   (a) Derive the virial theorem for a star in hydrostatic equilibrium.
   You may find the following equations useful;
   The equation of hydrostatic equilibrium,

$$\frac{dP}{dr} = -\frac{\rho(r)GM(r)}{r^2}, \tag{4}$$

   and the equation of potential energy

$$\Omega = -\int_0^{M_*}\frac{GM(r)}{r}dM. \tag{5}$$

   Also, the thermal energy density $\varepsilon$ in units of erg/cm$^3$ is given by: $\varepsilon = \frac{N}{V}\frac{3}{2}kT$.

4. **Stellar Evolution**

   (a) Sketch the Sun's evolution in the Hertzsprung-Russel (HR) diagram from the main sequence to a white dwarf. Use effective temperature and absolute luminosity for the axes, using the conventions of the HR diagram. Label the different stages.

   (b) The main sequence is not a single point on the star's evolutionary track. The Sun slowly increases in luminosity during its time on the Main Sequence. Explain why.

   (c) Describe the Sun's internal structure when it leaves the main sequence. What is this point called? Mark this point on your diagram.

   (d) What happens at the tip of the Red Giant Branch? Explain how this process happens.

5. **Stellar Structure**

   (a) What is the name of this equation?

$$\nabla_{\text{rad}} = \frac{3k_R}{64\pi r^2 g}\frac{P}{\sigma T^4}L(r) > \nabla_{\text{adi}} = \left(\frac{\gamma - 1}{\gamma}\right)$$

   (b) Explain why this equation is important, and describe two situations where this criterion holds.

6. **Radiative transfer**

   (a) State the main assumption for a grey atmosphere.

   (b) Using the radiative transfer equation, neglecting emissivity,

$$\frac{u}{\rho}\frac{dI_\nu(z,u)}{dz} = -k_\nu I_\nu(z,u), \tag{4}$$

   Show that

$$I_\nu(z) = I_\nu^0 e^{-k_\nu \rho z}. \tag{5}$$

7. **Atomic Lines**

   (a) Name three different line broadening mechanisms.

   (b) The Voigt-profile is the result of the convolution of two different broadening mechanisms.

      1. State which broadening mechanisms make up the Voigt-profile
      2. Indicate (i.e draw a sketch) how the profiles of these broadening mechanisms result in the Voigt-profile, clearly showing where their features are most prominent.

3. The hydrogen Balmer lines are typically much broader than other lines from other atoms in a stellar spectrum. Name and explain the mechanism responsible.

(c) The equivalent width is defined as the width of a hypothetical spectral line of rectangular shape that absorbs all radiation within its width and with the same total energy associated with the line,

$$W_\lambda = \int \left[1 - \frac{F_\lambda}{F_c}\right] d\lambda = \int [1 - R_\lambda] d\lambda = \int A_\lambda d\lambda. \tag{21}$$

1. Sketch the shape of a line and how its equivalent width relates to the line.
2. Determine the equivalent width, given the following line,

$$F_\lambda = \begin{cases} F_c \left[1 - \frac{2}{3}\cos\left(\frac{[\lambda - \lambda_0]\pi}{400\text{Å}}\right)\right], & \text{if } |\lambda - \lambda_0| \leqslant 200\text{Å}, \\ 0, & \text{otherwise.} \end{cases} \tag{7}$$

8. **Polytropic Models**

One of the well-known equations in stellar structure theory is the Lane-Emden equation:

$$\frac{1}{\xi^2}\frac{\mathrm{d}}{\mathrm{d}\xi}\left(\xi^2 \frac{d\theta(\xi)}{d\xi}\right) = -\theta^n(\xi), \tag{8}$$

where,

$$\xi = r\sqrt{\frac{4\pi G \rho_c^2}{(n+1)P_c}}, \tag{9}$$

a dimensionless quanity.

(a) The solution to this equation for n=0 is $\theta(\xi) = 1 - \frac{\xi^2}{6}$. Find $\xi_0$ such that $\theta(\xi_0) = 0$. At what location is $\xi_0$ defined for a star?

(b) A good approximation ($r \leq R_\odot/2$) of the inner workings of a star is given by a polytropic star with $n = 5$. Eq. (8) then has the solution:

$$\theta(\xi) = \frac{1}{\left(1 + \frac{\xi^2}{3}\right)^{1/2}}.$$

Can you reason why the approximation fails to describe a real star at $r > R_\odot/2$?

(c) Derive the stellar mass $M(r)$ for a polytropic star with $n = 5$, write your answer in terms of $\rho_c$, $r$ and $\xi$. (Hint: A polytropic density profile is written as $\rho(r) = \rho_c \theta^n(r)$)

**Handy integral**

$$\int \frac{x^2}{\left(\frac{x^2}{3} + 1\right)^{5/2}} dx = \frac{\sqrt{3}x^3}{(x^2+3)^{3/2}} + C$$