Practice session: Physics of Galaxies

Kapteyn Learning Community Sirius A

Give motivations and/or derivations for your answers.

1. **Conceptual starter questions**

   (a) Explain briefly Hubble's classification for late-type galaxies.

   (b) What does the Closed-Box galaxy evolution model assume?

   (c) In general, what changes in the received light from a galaxy if it contains considerable amount of dust compared to no dust?

   (d) What are all the main components of the Milky Way?

   (e) Explain differential rotation.

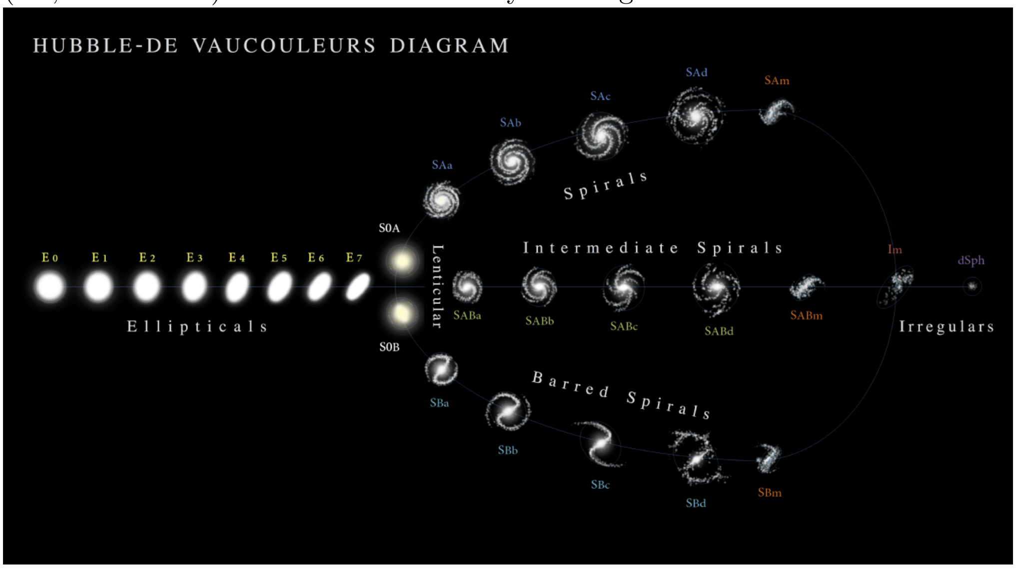
   (f) What does the morphology distribution of galaxies look like within a galaxy cluster compared to a more empty region in the Universe?

   (g) How does the spiral structure in a spiral galaxy not disappear due to the rotation?

   (h) Explain the concept of self-propagating star formation.

Solution:

(a) Depending if the galaxy has a bar or a spheroidal or a bulge it will have the classification (SB, SAB or SA). It is best described by an image has shown below:

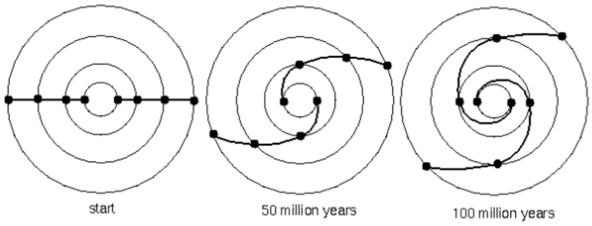


(b) It assumes no inflow of new gas, no outflow of metal-enriched gas, it starts of with only gas and no stars, and instant recycling of newly made metals (from supernovae) into the gas before new stars form.

(c) The received light will be fainter due to absorption by the dust and redder due to the wavelength dependent absorption and scattering of bluer light (shorter wavelengths), which is comparatively stronger then that of redder light.

(d) Components are the halo, thin disk, thick disk and the bulge.

(e) Differential rotation is that the rotation around the center is different depending on the distance from the center.

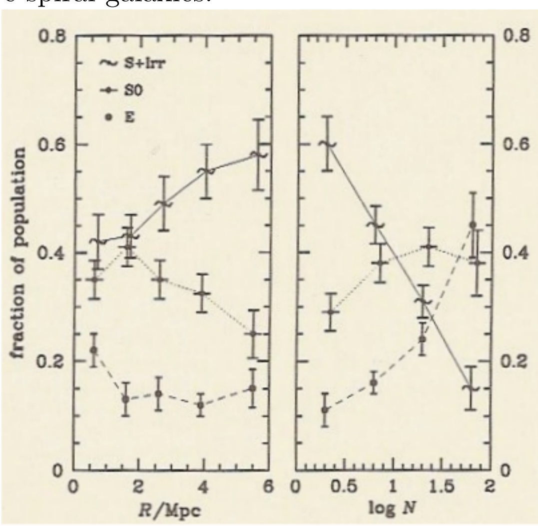


Differential rotation: stars near the center take less time to orbit the center than those farther from the center. Differential rotation can create a spiral pattern in the disk in a short time.

Differential rotation implies that stars exhibit both radial, tangential and vertical motions.

By studying these motions, we can infer the rotation curve of the Galaxy.

(f) In higher density regions there are more ellipticals while in less dense regions there are more spiral galaxies:

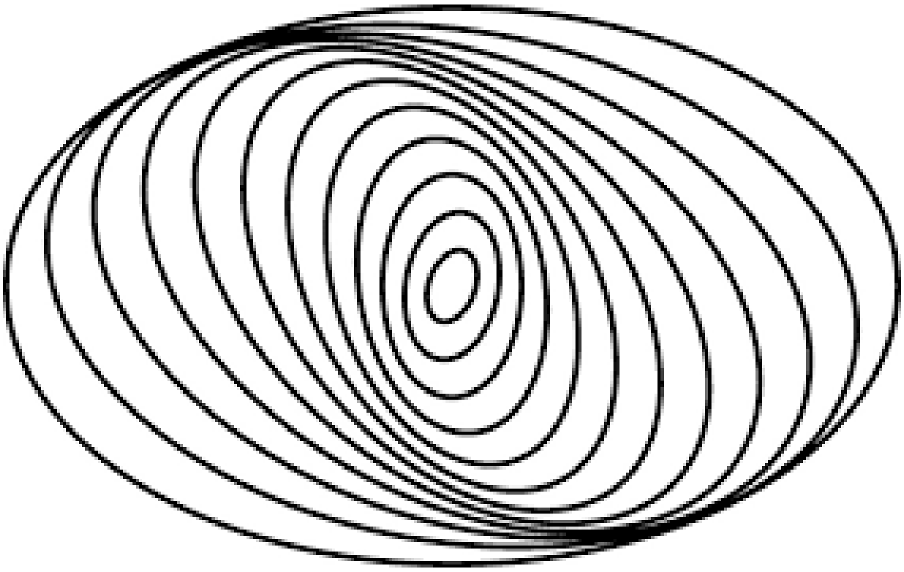


Fraction S goes up with distance from cluster centre

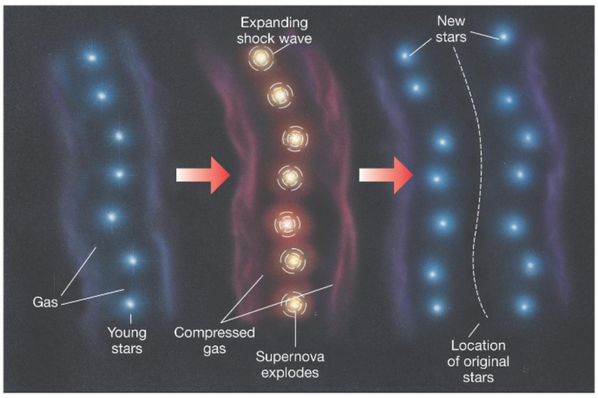
fraction of S goes down with surface density in cluster.

Dressler 1980

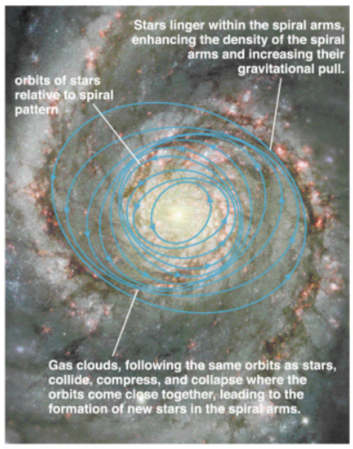
(g) Because they are density waves instead of pure rotations:



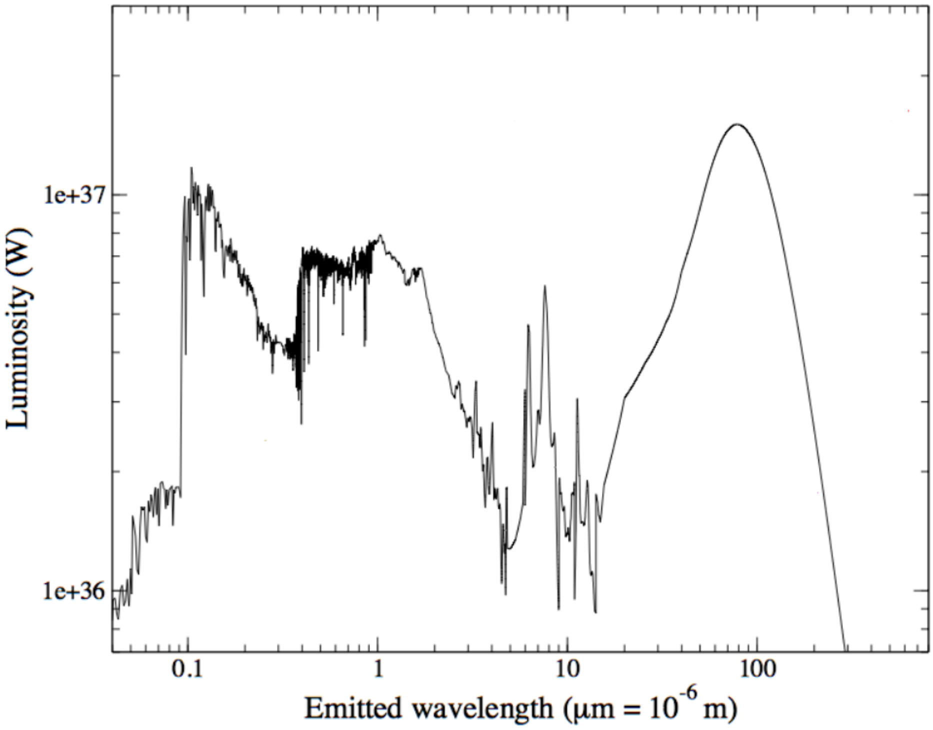
(h) Stars are formed during the shocks produced in the density waves like:



(a)



2. **Spectrum**
   This question will be mainly about the spectrum of a galaxy.



(a) What are the 4 components of the spectrum and which component of the galaxy contributes to this part of the spectrum

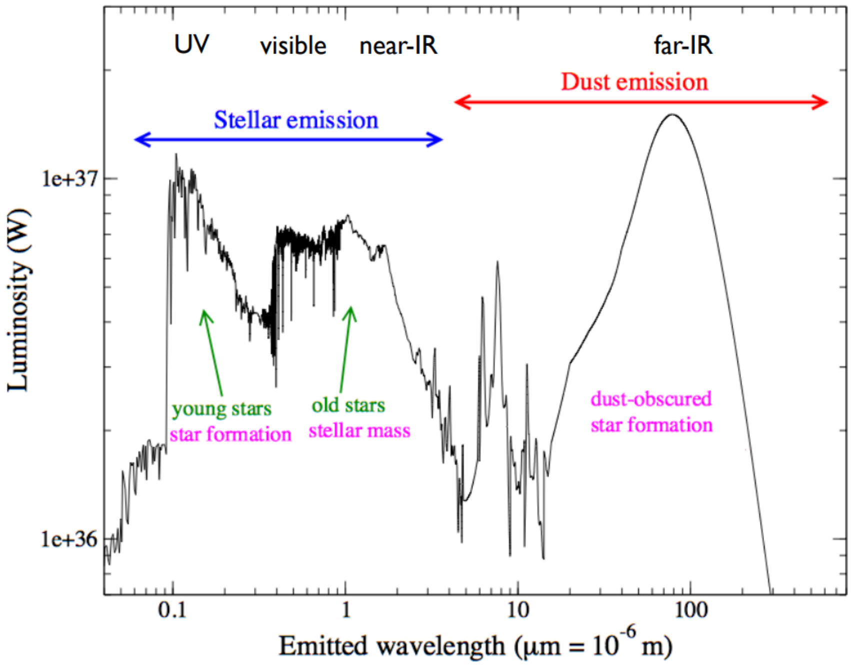
(b) Assume a single stellar population. Which stars dominate the luminosity of a galaxy a few million years, a few 50 million years and a billion years after that single star formation episode?

(c) How will the spectrum of a galaxy change if it were younger, i.e., you look at a galaxy at high redshift ($z > 5$)?

(d) Consider that this galaxy has a collision with an almost identical galaxy and this produces an enormous star formation peak after which star formation is quenched in the galaxy, How does this change the spectrum in the first few million years and how does the spectrum look like after a billion years.

Solution:

(a) The first part are young stars, the second are older red stars and the middle part are the PAH and the most right is the dust:



(b) The most massive O and B stars dominate in the first few million years, in the 50 million years the low mass B and A stars dominate, while at a billion years only the low mass red stars will contribute to the spectrum.

(c) At a higher redshift there is less dust, which means that the dust curve is lower.

(d) In the first part the young stars will dominate while later the low mass stars will start to dominate and the dust will be higher at the second time due to SNe of the new young stars.

3. **Spherical galaxy**
Consider a spherical galaxy for which the galaxy mass density can be described as:

$$\rho = \rho_0 \exp\left(-\frac{r^2}{2R_{\rm gal}^2}\right). \tag{1}$$

with $\rho_0 = 20\ M_\odot/\mathrm{pc}^3$ and $R_{\rm gal} = 2$ kpc.

(a) What is the total mass of the galaxy? Your answer should be a number. (Use $\int_0^\infty y^2 e^{-y^2} = \frac{\sqrt{\pi}}{4}$)

(b) What is the ratio between the circular velocities of two stars, one at a distance $r_1 = 0.5$ kpc from the centre, and the other at $r_2 = 1.0$ kpc from the centre? (Use $\int_0^b \mathrm{d}y\ y^2 e^{-y^2} = \frac{1}{4}\left(\sqrt{\pi}\mathrm{erf}(b) - 2be^{-b^2}\right)$, $\mathrm{erf}(1/4\sqrt{2}) \approx 0.197$ and $\mathrm{erf}(1/2\sqrt{2}) \approx 0.383$)

(c) The average mass-to-light ratio of the galaxy in the V band is $M/L_V \approx 2.5\ \mathrm{M}_\odot/\mathrm{L}_\odot$. If this galaxy were located at distance of $D = 200$ Mpc, what would its apparent V-band magnitude (m_V) be? Give your answer in AB magnitudes (Use that the absolute V-band magnitude of the sun is 4.862).

(d) Close to this galaxy, there is a galaxy that is 20 arcseconds on the sky, what is the physical size of this galaxy?

Solution:

(a) For this we need to solve a relatively easy integral:

$$M = 4\pi\rho_0 \int_0^\infty \mathrm{d}r\, r^2 \exp\left(-\frac{r^2}{2R_{gal}^2}\right), \tag{2}$$

$$= 4\pi\rho_0 \left(\sqrt{2}R_{gal}\right)^3 \int_0^\infty \mathrm{d}y\, y^2 e^{-y^2}, \tag{3}$$

$$= 4\pi\rho_0 \left(\sqrt{2}R_{gal}\right)^3 \frac{\sqrt{\pi}}{4}, \tag{4}$$

$$= (2\pi)^{3/2} \frac{20\ \mathrm{M}_\odot}{\mathrm{pc}^3} \left(2\cdot 10^3\ \mathrm{pc}\right)^3, \tag{5}$$

$$= 2.5\cdot 10^{12}\ \mathrm{M}_\odot. \tag{6}$$

(b) The rotation velocity is given by:

$$V^2(r) = \frac{GM(r)}{r}. \tag{7}$$

We know the mass is given by:

$$M(r) = 4\pi\rho_0 \int_0^r \mathrm{d}r\, r^2 \exp\left(-\frac{r^2}{2R_{gal}^2}\right), \tag{8}$$

$$= 4\pi\rho_0 2^{3/2} R_{gal}^3 \int_0^{r\sqrt{2}R_{gal}} \mathrm{d}y\, y^2 e^{-y^2}. \tag{9}$$

This means that the fraction between the two velocities is given by:

$$\frac{V(r_{min})}{V(r_{max})} = \sqrt{\frac{\left(\sqrt{\pi}\mathrm{erf}\left(\frac{1}{4\sqrt{2}}\right) - 2\cdot\frac{1}{4\sqrt{2}}e^{-1/32}\right)}{\left(\sqrt{\pi}\mathrm{erf}\left(\frac{1}{2\sqrt{2}}\right) - 2\cdot\frac{1}{2\sqrt{2}}e^{-1/8}\right)}2} = 0.49 \tag{10}$$

(c) Because of the mass-to-light ratio the total luminosity is $10^{12}\mathrm{L}_{\odot,V}$. This means that the flux at earth is given by: $f = L/4\pi D^2$, that means we obtain:

$$M = -2.5\log(10^{12}) + 4.862 = -25.138 \tag{11}$$

$$m = -25.138 + 5\log(200\cdot 10^5) = 11.367 \tag{12}$$

(d) The size of the galaxy can be calculated as $s = 20/206264 \cdot 200 \cdot 10^3$ kpc $= 19.4$ kpc. Note that $60 \cdot 60 \cdot \frac{180}{\pi} = 206264$.

4. **Astronomical distances**

This question surrounds astronomical distances and how to interpret different answers.

(a) A galaxy is observed with a redshift of 0.11. At what distance is the galaxy located according to the Hubble law?

(b) The Galaxy has an absolute magnitude of -19.5. what is the expected apparent magnitude

(c) The apparent magnitude of the Galaxy is found to be 1 magnitude higher then expected. Assuming the Absolute magnitude is correct what could be the reason for this error.

(d) Using the magnitudes calculate the actual distance to the galaxy

(e) Calculate the peculiar velocity(velocity that deviates from Hubble law) and the actual redshift due to Hubble expansion.

Solution:

(a) $0.11c \approx 33000\text{km/s} \implies 33000/H_0 = 471Mpc$

(b) $m = -5 + 5\log_{10} d + M = 18.9$

(c) The galaxy appears fainter then calculated, the galaxy must thus be at a greater distance then calculated from the Hubble law. This means the redshift is not just from Hubble expansion but there is also a radial velocity towards us, this decreases the redshift.

(d) $d = 10^{\frac{m-M+5}{5}} = 759$ Mpc

(e) The velocity using the Hubble law would be $v_{H_0} = H_0 \cdot d = 53130$ km/s and the redshift $v_{H_0}/c = z = 0.1771$. (This means its moving towards us at $\Delta v = 53130 - 33000 \approx 20000$ km/s which is pretty flippin fast)

5. **The Initial Mass Function (IMF)**

The IMF of a galaxy governs the initial distribution of stellar mass among a stellar population, i.e., the number of stars of a particular mass. Consider a stellar population with the Salpeter IMF $1.72 \times 10^4\,(\text{M}/\text{M}_\odot)^{-2.35}$ and a stellar mass range of $(0.1 - 100)\text{M}_\odot$.

(a) At the time the stellar population is born, what percentage of stars have masses in the range $(0.1 - 1.0)\text{M}_\odot$?

(b) What is the total mass of this stellar population?

(c) What can you say about which stars dominate the total mass, and which stars dominate the total luminosity of this stellar population based on the IMF? (Assume luminosity $L \propto M^{3.5}$)

Solution:

(a) The number of stars in a given mass range can be obtained from the IMF as follows:

$$\begin{aligned} N &= \int_{m_l}^{m_u} \xi(m)dm \\ &= \int_{m_l}^{m_u} \xi_0\, m^{-2.35} dm \\ &= \frac{\xi_0}{-1.35} m^{-1.35}\Big|_{m_l}^{m_u} \\ &= \frac{\xi_0}{1.35}(m_l^{-1.35} - m_u^{-1.35}), \end{aligned} \tag{13}$$

where $m = \mathrm{M}/\mathrm{M}_\odot$, m_l, and m_u are the bounds of the lower and upper mass range, respectively.
The percentage of stars within $(0.1-1.0)\mathrm{M}_\odot$ for this stellar population at the time it is born is then:

$$\frac{(0.1-1.0)\mathrm{M}_\odot}{(0.1-100)\mathrm{M}_\odot} = \frac{0.1^{-1.35} - 1.0^{-1.35}}{0.1^{-1.35} - 100^{-1.35}} \approx 96\,\%. \tag{14}$$

(b) The total mass of the stellar population is the number of stars per unit mass (i.e., the IMF) multiplied by the mass of each star, integrated over the entire mass range:

$$\begin{aligned} \text{Mass} &= \int_{0.1}^{100} \mathrm{M}\xi(m)dm\ [\mathrm{M}_\odot] \\ &= \xi_0 \int_{0.1}^{100} \mathrm{M}\cdot\mathrm{M}^{-2.35}\, d\mathrm{M}\ [\mathrm{M}_\odot] \\ &= \xi_0 \int_{0.1}^{100} \mathrm{M}^{-1.35}\, d\mathrm{M}\ [\mathrm{M}_\odot] \\ &= \frac{\xi_0}{-0.35}\mathrm{M}^{-0.35}\Big|_{0.1}^{100}\ [\mathrm{M}_\odot] \\ &= \frac{\xi_0}{0.35}(0.1^{-0.35} - 100^{-0.35})\ [\mathrm{M}_\odot] \\ &\approx 10^5\,\mathrm{M}_\odot, \end{aligned} \tag{15}$$

where the destinction between m and M is important.

(c) Because of the negative exponent after integrating to obtain the total mass, the lower mass stars have considerably more weight than the high mass stars, $100^{-0.35} \ll 0.1^{-0.35}$. When calculating the total luminosity using $\mathrm{L} = \int L\,\xi(m)dm \propto \int m^{3.5}\ m^{-2.35}dm \propto \int m^{1.15}dm \propto m^{2.15}$, note that exponent is positive and $100^{2.15} \gg 0.1^{2.15}$. In short, the total mass is dominated by the lower mass stars, and the total luminosity by the high mass stars.

6. **A Galaxy with no Dark Matter?**
In 2018, a team of astronomers reported their discovery of a galaxy with much less dark matter

than the galaxy evolution model predicted (van Dokkum et al. 2018, Nature).

This galaxy, named NGC 1052-DF2, is located close to the elliptical galaxy NGC 1052 ($D = 20$ Mpc from the Sun) in the sky. The shape of NGC 1052-DF2 resembles an ellipse with a semi-major axis (a) of 22.6" and $a/b = 0.85$.

Half of the total light from the galaxy comes from within this ellipse and the mean surface brightness within the ellipse is about 24.7 mag arcsec^{-2}.

(a) Calculate the total apparent magnitude of this galaxy.

(b) The team suggested the galaxy is a companion of NGC 1052. Determine the total mass of stars in NGC 1052-DF2, assuming it has a mass to light ratio ($\frac{M/M_\odot}{L/L_\odot}$) of 2.0.

(c) The team identified 10 globular clusters in NGC 1052-DF2 with a mean galactocentric distance of 78.4". They also measured the velocity dispersion of these clusters to be not more than 8.4 km/s. Estimate the dynamical mass of this galaxy. For simplicity, assume the mass distribution in the galaxies is uniform and is spherically symmetric.

(d) This discovery was challenged by other groups (Kroupa et al., Nature, 2018, Truijlo et al., MNRAS, 2018), who claimed that NGC 1052-DF2 is not a satellite of NGC 1052, and it is located at a much smaller distance to us. Show why a smaller distance would weaken the assertion of the dark matter deficiency in NGC 1052-DF2.

Solution:

(a) The area of the half-light ellipse is

$$A = \pi ab = \pi(22.6)(19.21) \approx 1.364 \times 10^3 \text{ arcsec}^2.$$

If the mean surface brightness within this ellipse is μ, then the apparent magnitude of the light *inside* the ellipse is

$$m_{\frac{1}{2}} = \mu - 2.5 \log_{10} A.$$

$$m_{\frac{1}{2}} \approx 24.7 - 2.5(3.135) = 24.7 - 7.84 = 16.86.$$

Since this is half the total light, the total magnitude is brighter by

$$2.5 \log_{10} 2 \approx 0.753.$$

Therefore

$$m_{\text{tot}} = m_{\frac{1}{2}} - 2.5 \log_{10} 2 \approx 16.86 - 0.753 = 16.11.$$

(b) Assuming the galaxy is at the same distance as NGC 1052,

$$D = 20 \text{ Mpc} = 2.0 \times 10^7 \text{ pc}.$$

The distance modulus is

$$m - M = 5 \log_{10}\left(\frac{D}{10 \text{ pc}}\right) = 5 \log_{10}(2.0 \times 10^6).$$

So, we get

$$m - M = 5(6.301) = 31.51.$$

Thus the absolute magnitude is

$$M = m_{\rm tot} - (m - M) = 16.11 - 31.51 = -15.40.$$

Using the Sun's absolute magnitude in the V-band,

$$M_{V,\odot} \approx 4.83,$$

the luminosity is

$$\frac{L}{L_\odot} = 10^{-0.4(M - M_{V,\odot})} = 10^{-0.4(-15.40-4.83)} = 10^{8.09} \approx 1.24 \times 10^8.$$

With mass-to-light ratio

$$\frac{M_\star/M_\odot}{L/L_\odot} = 2.0,$$

the stellar mass is

$$M_\star = 2.0\,L \approx 2.0(1.24 \times 10^8)M_\odot \approx 2.5 \times 10^8\,M_\odot.$$

(c) Take the characteristic radius of the globular clusters:

$$\theta = 78.4''.$$

Using the small-angle formula,

$$R = D\theta,$$

with θ in radians:

$$\theta = \frac{78.4}{206265} \approx 3.80 \times 10^{-4} \text{ rad}.$$

So

$$R = (2.0 \times 10^7 \text{ pc})(3.80 \times 10^{-4}) \approx 7.60 \times 10^3 \text{ pc} = 7.60 \text{ kpc}.$$

Assuming spherical symmetry and using a virial-style estimate,

$$M_{\rm dyn} \sim \frac{R\sigma^2}{G},$$

where

$$\sigma = 8.4 \text{ km s}^{-1}, \qquad G = 4.302 \times 10^{-6}\ \frac{\text{kpc}\,(\text{km/s})^2}{M_\odot}.$$

Then

$$M_{\rm dyn} \approx \frac{(7.60)(8.4)^2}{4.302 \times 10^{-6}}\,M_\odot.$$

We obtain

$$M_{\rm dyn} \approx \frac{536.3}{4.302 \times 10^{-6}}\,M_\odot \approx 1.25 \times 10^8\,M_\odot.$$

This is comparable to, or even somewhat smaller than, the stellar mass estimate, so there is little room for a large dark matter component.

(d) Let the true distance be smaller than 20 Mpc.

From the observed flux,

$$L \propto D^2,$$

so

$$M_\star \propto D^2.$$

The observed velocity dispersion σ does not depend on distance, but the physical radius does:

$$R = \theta D \propto D.$$

Therefore

$$M_{\rm dyn} \sim \frac{R\sigma^2}{G} \propto D.$$

So as distance decreases,

$$M_\star \propto D^2 \quad \text{falls faster than} \quad M_{\rm dyn} \propto D.$$

Hence

$$\frac{M_{\rm dyn}}{M_\star} \propto \frac{D}{D^2} = \frac{1}{D},$$

which *increases* when D becomes smaller.

That means if NGC 1052-DF2 is closer than 20 Mpc, then for the same observed brightness and angular size:

- the inferred stellar mass becomes much smaller,
- the inferred dynamical mass decreases only linearly,
- therefore the discrepancy $M_{\rm dyn} > M_\star$ becomes larger.

So a smaller distance makes the galaxy appear *less* deficient in dark matter, weakening the original claim.

7. **Kinematic classes of early-type galaxies**
Two early-type galaxies are observed with an integral-field spectrograph, yielding the following measurements:

	Galaxy C	Galaxy D
Rotation velocity V	140 $\mathrm{km\,s^{-1}}$	35 $\mathrm{km\,s^{-1}}$
Velocity dispersion σ	110 $\mathrm{km\,s^{-1}}$	260 $\mathrm{km\,s^{-1}}$
Ellipticity ε	0.32	0.06

(a) Using the proxy for the λ_R estimator,

$$\lambda_R \approx \frac{V}{\sqrt{V^2+\sigma^2}},$$

classify each galaxy as a fast or slow rotator, given that the dividing line is $\lambda_R = 0.1$.

(b) Galaxy C is significantly flattened, while Galaxy D is nearly round. Explain qualitatively why this is consistent with their kinematic classification.

(c) Which of the two galaxies is more likely to have formed through repeated dry mergers? Briefly justify your answer.

(d) Fast rotators are often described as being more "disk-like" than slow rotators. Explain what is meant by this statement in terms of the balance between ordered motion and random motion.

Solution:

(a) For Galaxy C:

$$\lambda_R \approx \frac{140}{\sqrt{140^2 + 110^2}} = \frac{140}{\sqrt{19600 + 12100}} = \frac{140}{\sqrt{31700}} \approx \frac{140}{178.0} \approx 0.79.$$

For Galaxy D:

$$\lambda_R \approx \frac{35}{\sqrt{35^2 + 260^2}} = \frac{35}{\sqrt{1225 + 67600}} = \frac{35}{\sqrt{68825}} \approx \frac{35}{262.3} \approx 0.13.$$

Since both values are greater than the dividing line $\lambda_R = 0.1$, both galaxies would formally be classified as fast rotators using this simple proxy. However, Galaxy C is clearly a strong fast rotator, while Galaxy D lies very close to the boundary and would be considered only marginally consistent with that classification.

(b) Galaxy C has a relatively high ellipticity ($\varepsilon = 0.32$), which means that it is noticeably flattened. This is consistent with a fast-rotating system, because rotational support tends to flatten a galaxy.

Galaxy D is nearly round ($\varepsilon = 0.06$), which suggests that rotation is not the dominant form of support. Nearly round early-type galaxies are more often associated with systems supported by random stellar motions rather than ordered rotation. This makes Galaxy D much less obviously disk-like, even though its simple λ_R estimate lies just above the formal dividing line.

(c) Galaxy D is more likely to have formed through repeated dry mergers. Dry mergers tend to disrupt ordered rotation and increase the importance of random stellar motions, producing galaxies with high velocity dispersion, low ellipticity, and weak rotational support. Galaxy D has a very large velocity dispersion and a very small ellipticity, which are characteristic of a merger-built slow-rotator-like system.By contrast, Galaxy C has strong rotation and a flattened shape, which are more consistent with a galaxy that has retained a significant disk component.

(d) Saying that fast rotators are more "disk-like" means that their stars have a larger component of ordered rotational motion compared with random motion. In a disk-like system, many stars move on roughly circular orbits around the galaxy centre, producing a substantial rotation velocity V. In contrast, in a slow rotator, the stellar motions are more disordered, so the velocity dispersion σ is more important than the net rotation. Thus, fast rotators are supported mainly by ordered motion, while slow rotators are supported mainly by random motion.