Practice Session — Physics Laboratory: Skills

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Give motivations and/or derivations for your answers.

Using the integral table of a Gaussian and the table for the cumulative χ^2 is allowed.

1. **Observing Decays**

   The decay of a sample of radioactive material is measured and an *average* rate of 3.5 decays per second is observed.

   Calculate what the probability P is of observing exactly 5 decays in 1 second, given the following:

$$f(k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

Solution: Calculate what the probability P is of observing exactly 5 decays in 1 second.
Probability $P = f(k) = \frac{\lambda^k}{k!} e^{-\lambda}$ with $\lambda = 3.5$ and $k = 5$ so $P = f(k) = \frac{3.5^5}{5!} e^{-3.5}$ so $\boxed{P = 13\%}$.

2. **Measuring Brightness**

   We are measuring the brightness of a faint star, i.e. the rate of incoming photons, which we calculate to be $N = 4 \cdot 10^8$ photons under one second of exposure time.

   (a) **Compute** the uncertainty in the number of detected photons, assuming that it is a Poisson random variable!

   (b) **Reason** why we still make long exposures with telescopes, even though the uncertainty in the number of detected photons is increasing with exposure time! *Hint: Think of the relative uncertainty in brightness.*

Solution:

(a) The uncertainty in a Poisson process is $\sqrt{N}$, so that the uncertainty in the number of detections is

$$\boxed{\sqrt{N} = 2 \cdot 10^4 \text{ photons}}$$

(b) The relative uncertainty in the photon counts is given by

$$\delta N = \frac{\sigma_N}{|N|} = \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}} = \frac{1}{\sqrt{\lambda \cdot t_{\text{exp}}}}$$

Clearly, as we increase t_{exp}, the ***relative* uncertainty diminishes**. Also, one can see that a brighter star (with a greater rate parameter λ) has a less uncertain brightness.

3. **Proper Precision**

   Rewrite the following results using the correct notation (proper precision and number of significant digits)

   (a) $f = 31.41$ Hz ± 0.5 Hz

   (b) $F = 3.141 \frac{\text{kW}}{\text{m}^2} \pm 59 \frac{\text{W}}{\text{m}^2}$

   (c) $S = 3.1$ μJy ± 41 nJy

   (d) $\Omega = 314.159$ sr ± 2 msr

(e) $E = 3141$ kerg ± 0.592 Merg

Solution:
Any reasonable power of 10 or unit is good if it can be converted to the example solutions.

(a) $f = 31.41$ Hz ± 0.5 Hz
$\boxed{f = 31.4 \pm 0.5 \text{ Hz}}$ or $\boxed{f = (3.141 \pm 0.05) \cdot 10^1 \text{ Hz}}$

(b) $F = 3.14 \frac{\text{kW}}{\text{m}^2} \pm 59 \frac{\text{W}}{\text{m}^2}$
$\boxed{F = 3.14 \pm 0.06 \frac{\text{kW}}{\text{m}^2}}$ or $\boxed{F = (3.14 \pm 0.06) \cdot 10^3 \frac{\text{W}}{\text{m}^2}}$

(c) $S = 3.1$ μJy ± 41 nJy
$\boxed{S = 3.10 \pm 0.05 \text{ } \mu\text{Jy}}$ or $\boxed{S = (3100 \pm 5) \cdot 10^1 \text{ nJy}}$ or $\boxed{S = (310 \pm 5) \cdot 10^{-2} \text{ } \mu\text{Jy}}$

(d) $\Omega = 314.159$ sr ± 2 msr
$\boxed{\Omega = 314159 \pm 2 \text{ msr}}$

(e) $E = 3141$ kerg ± 0.592 Merg
$\boxed{E = 3.1 \pm 0.6 \text{ Merg}}$

4. **Types of uncertainties**

You have a multimeter that measures the voltage on a resistor, set to mV sensitivity range. The currents suitable for the multimeter are in the mA range. You see the following value on the screen of the apparatus:



(a) **Reason** which type of uncertainty you can infer from the information available. You can assume that the digits are not changing over time (see 'HOLD' state in the top right corner).

(b) **Estimate** this error using the image above.

(c) After releasing the HOLD state, the last digit of the measured value starts fluctuating. **Indicate** which type of error this represents.

(d) **Explain** the following: if there is (or would be) a systematic error in the measurement, what effect does it have on your measured value?
For example, you measure the same system with two more multimeters and both of them show a value around $4mV$, while reconnecting this one still gives a $\approx 5mV$ result.

Solution:

(a) **Reason** which type of uncertainty you can infer from the information available. You can assume that the digits are not changing over time (see 'HOLD' state in the top right corner).

This is an assigned error stemming from the limited sensitivity of the apparatus.

(b) **Estimate** this error using the image above.

Since the voltage is in the mV range, the last digit shown has powers of 10^{-6}. This is a

good estimate for the assigned error of the voltage, with units of V. Therefore: $\Delta V_{\text{assigned}} = 10^{-6}\ \text{V} = 10^{-3}\ \text{mV}$

(c) After releasing the HOLD state, the last digit of the measured value starts fluctuating. **Indicate** which type of error this represents.

This is the reading error, a type of statistical error.

(d) **Explain** the following: if there is (or would be) a systematic error in the measurement, what effect does it have on your measured value?
For example, you measure the same system with two more multimeters and both of them show a value around $4mV$, while reconnecting this one still gives a $\approx 5mV$ result.

The systematic error would not influence the uncertainty in the measurement, however it can make the measured value itself shift towards lower or higher values, causing a significant deviation from the 'true' value. A perfect example for systematic error is a measuring tape: if the scale is more dense than it is supposed to be, it introduces a systematic error.

5. **Escaping With Your Spaceship**
The escape velocity v is the minimum velocity required to leave a planet with your spaceship and is such that it overcomes the pull of gravity. It is given by:

$$v = \sqrt{\frac{2Gm}{r}}$$

with escape velocity v, planet radius r, planet mass m, and the gravitational constant G.

(a) **Give** the (SI) units of G. Show at least one intermediate step.

(b) **Calculate** the relative error and the absolute error in v given that a space shuttle with some velocity v tries to escape a planet of $m = (1234.5 \pm 0.9) \cdot 10^{20}$ kg, with $G = 6.67430 \cdot 10^{-11}$ (in the appropriate units) and $r = (4534.2 \pm 0.7) \cdot 10^{2}$ m.

(c) **Write** the final result in the correct notation $v = ... \pm ....$

Solution:

(a) **Give** the (SI) units of G. Show at least one intermediate step.

$$v = \sqrt{\frac{2Gm}{r}} \Rightarrow \quad G = \frac{v^2 r}{2m} \Rightarrow \quad \boxed{[G] = \frac{\text{m}^2}{\text{s}^2}\frac{\text{m}}{\text{kg}} = \frac{\text{m}^3}{\text{s}^2\ \text{kg}}}$$

(b) **Calculate** the relative error and the absolute error in v given that a space shuttle with some velocity v tries to escape a planet of $m = (1234.5 \pm 0.9) \cdot 10^{20}$ kg, with $G = 6.67430 \cdot 10^{-11}$ and $r = (4534.234 \pm 0.762) \cdot 10^{2}$ m.

$$v = \sqrt{\frac{2Gm}{r}} = \boxed{6.029 \cdot 10^{3}\ \text{m/s}}$$

By using the Gaussian propagation of uncertainties formula:

$$\Delta v = \sqrt{\left(\frac{\partial v}{\partial m}\Delta m\right)^2 + \left(\frac{\partial v}{\partial r}\Delta r\right)^2} = \sqrt{\left(\frac{G}{vr}\Delta m\right)^2 + \left(\frac{Gm}{vr^2}\Delta r\right)^2} = 2.246\,\frac{\text{m}}{s}$$

so $\delta v = \frac{\Delta v}{v} = \boxed{5.553 \cdot 10^{-7}}$

(c) **Write** the final result in the correct notation $v = ... \pm ....$

$\boxed{v = (6.029 \pm 0.003) \cdot 10^{3}\ \text{m/s}}$

6. **Rowing Competition**
In lightweight rowing for men, the average weight of the team members is not allowed to be higher than 70 kg. The weight of each team member is not allowed to be more than 72.5 kg. *Robert* ($[68.1 \pm 0.3]$ kg) and *Hans* ($[72.1 \pm 0.4]$ kg) want to participate in a lightweight rowing competition. They will measure their weight. Hans weight is also determined with another scale, giving $[72.4 \pm 0.3]$ kg.

(a) **Calculate** the weighted average and weighted error of the weights for Hans.

Robert and Hans decide to take part in the competition. The crew of the competition knows nothing about statistics and random errors and therefore performs only one measurement of each of the team members' weights, finding the following results: Robert 67.9 kg, Hans 72.3 kg. By calculation of the average weight the crew decides to disqualify the team.

(b) **Calculate** the average weight the crew finds.

Robert is not to be caught for one hole. He decides to interchange his everyday shoes for socks and sandals. Assume that on average socks and sandals are 0.3 kg lighter than normal shoes.

(c) **Reason** whether this changing of shoes is enough to be allowed to participate in the competition.

Solution:

(a) **Calculate** the weighted average and weighted error of the weights.

$$w_1 = \frac{1}{s_1^2} = \frac{1}{0.3^2} = 11.11 \qquad w_2 = \frac{1}{s_2^2} = \frac{1}{0.4^2} = 6.25$$

$$M = \frac{w_1 M_1 + w_2 M_2}{w_1 + w_2} = \frac{11.11 \cdot 68.1 + 6.25 \cdot 72.1}{11.11 + 6.25} = 72.29 \text{ kg} \approx \boxed{72.3 \text{ kg}}$$

$$\frac{1}{s_M^2} = \frac{1}{s_1^2} + \frac{1}{s_2^2} \qquad \boxed{s_M = 0.24 \text{ kg} \approx 0.3 \text{ kg}}$$

Robert and Hans decide to take part in the competition. The crew of the competition knows nothing about statistics and random errors and therefore performs only one measurement of each of the team members' weights, finding the following results: Robert 67.9 kg, Hans 72.3 kg. By calculation of the average weight the crew decides to disqualify the team.

(b) **Calculate** the average obtained by the crew.

$$M_{\text{average}} = \frac{1}{N} \sum_{i=1}^{N} M_i = \frac{67.9 + 72.3}{2} = \boxed{70.1 \text{ kg}}$$

Robert is not to be caught for one hole. He decides to interchange his everyday shoes for sandals. Assume that on average sandals are 0.3 kg lighter than normal shoes.

(c) **Reason** whether this changing of shoes is enough to be allowed to participate in the competition.
Robert's weight is now 67.6 kg, resulting in an average weight of 69.95 kg, which would allow participation in the competition.

7. **The Mass Of The Milky Way**
Two students have bought *Sint Maarten* candy, but no children came to their home to sing, so they are left with a bag full of mini Milky Way bars. They start measuring the mass of the bars, to see whether all bars are equally heavy. The results are as follows: $m_1 = 16.6$ g, $m_2 = 16.8$ g, $m_3 = 16.5$ g, $m_4 = 16.0$ g, $m_5 = 16.9$ g

(a) **Calculate** the average mass $\bar{m}$.

(b) **Calculate** the best estimate for the standard deviation in these measurements.

(c) **Determine** the error s_m, the best estimate for the standard error in the mean.

(d) **Reason** how many measurements should be taken in order to reduce the error in these measurements by a factor 3.

Solution:

(a) **Calculate** the average mass $\bar{m}$.

$$\bar{m} = \frac{1}{N}\sum_{i=1}^{N} m_i = \frac{1}{5}\left(16.6 + 16.8 + 16.5 + 16.0 + 16.9\right) = \boxed{16.56 \text{ g}}$$

(b) **Calculate** the best estimate for the standard deviation in these measurements.

$$s_m = \sqrt{\frac{\sum_{i=1}^{N}(m_i - m)^2}{N-1}} = \sqrt{\frac{(0.04)^2 + (0.24)^2 + (-0.06)^2 + (-0.56)^2 + (0.34)^2}{4}} \approx 0.3507 \text{ g} = \boxed{0.4 \text{ g}}$$

(c) **Determine** the error s_m, the best estimate for the standard error in the mean.

$$s_{\bar{m}} = \frac{s_m}{\sqrt{N}} \approx \frac{0.3507 \text{ g}}{\sqrt{5}} \approx 0.16 \text{ g} = \boxed{0.2 \text{ g}}$$

(d) **Reason** how many more measurements should be taken to reduce the error in these measurements by a factor 3.
The error in the mean decreases by a factor of $\sqrt{N}$, so in order for the error to be 3 times as small, the number of measurements should be $3^2 = 9$ times as large. This yields $5 \cdot 9 = \boxed{45}$ measurements.

8. **Physics Lab Grades**

q	$n \pm \Delta n$
1.00	10 ± 2
2.00	25 ± 2
3.00	36 ± 2
4.00	39 ± 2

A series of 4 observations is given in the table above, here q is the grade of a student on their Physics Lab: Skills exam and n is the number of students with this grade. The error in q is negligible. A straight line $n = a \cdot q + b$ is fitted to these observations with the help of Tailor. It is found that $a = [10 \pm 2]$ and $b = [3 \pm 6]$.

(a) **Calculate** the value of χ^2 for this fit.

(b) Use the χ^2 test to **argue** whether this linear fit is acceptable on the 10-90% probability level.

Solution:

(a) **Calculate** the value of χ^2 for this fit.

By definition, $\chi^2 = \sum_{i=1}^{N} \left(\frac{y_i - f(x_i)}{\Delta y_i} \right)^2$ which, for this problem, reads

$$\chi^2 = \sum_{i=1}^{4} \left(\frac{n_i - aq_i - b}{\Delta n} \right)^2 = \boxed{9.5}$$

(b) Use the χ^2 test to **argue** whether this linear fit is acceptable on the 10-90% probability level.

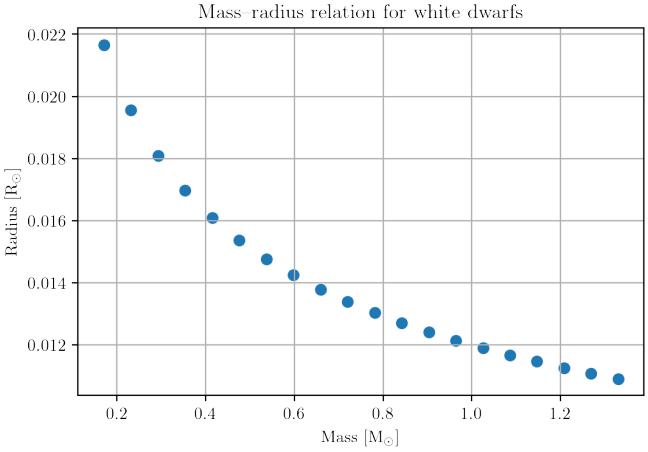
We are fitting two parameters, to the degrees of freedom are $\nu = N - n_{\text{param}} = 2$, for which the threshold values for the cumulative χ^2 are 0.211 and 4.605 for 10% and 90%, respectively. As χ^2 is significantly greater than the upper limit, the data is underfitted, the linear fit is **not acceptable**.

9. **White Dwarfs**

Interestingly, white dwarf stars with smaller radii have greater masses. The relation between these variables can be cast in the form: $R = kM^{\beta}$

where M and R is measured in solar masses and radii, respectively.

Using the graph below, **estimate** the constants k and β! What is the dimension of k? You do not have to estimate uncertainties.



Solution: We choose two nicely placed data points with easily readable coordinates, (x_1, y_1) and (x_2, y_2).

We know $y_1 = kx_1^{\beta}$ and $y_2 = kx_2^{\beta}$, so dividing the equations yields

$$\frac{y_2}{y_2} = \left(\frac{x_2}{x_1} \right)^{\beta} \implies \ln(y_2/y_1) = \beta \ln(x_2/x_1) \implies \beta = \frac{\ln(y_2/y_1)}{\ln(x_2/x_1)}$$

Choosing the fifth point with $(M, R) = (0.4\ \mathrm{M}_{\odot}, 0.016\ \mathrm{R}_{\odot})$ and an interpolated point at $(M, R) = (1.0\ \mathrm{M}_{\odot}, 0.012\ \mathrm{R}_{\odot})$, using the relation for β gives:

$$\boxed{\beta = -0.314}$$

which is close to the theoretical value of $\beta = -1/3$.

Finally, using the second data point, we have

$$R = kM^{\beta} \implies k = RM^{-\beta} \implies \boxed{k = 0.012}$$

10. **Unit Conversion**
Convert the following data into the desired units! It is given that $1\ \text{pc} = 3.086 \cdot 10^{16}\ \text{m}$.

Value	Convert to:
$5.67 \cdot 10^{-8}\ \text{W m}^{-2}\text{K}^{-4}$	$\text{mW cm}^{-2}\ \text{nK}^{-4}$
$1.055 \cdot 10^{-34}\ \text{Js}$	$\mu\text{g km}^2\ \text{h}^{-1}$
$70\ \text{km s}^{-1}\text{Mpc}^{-1}$	year^{-1}

Solution: We are asked to convert the following quantities into the specified units. It is given that

$$1\ \text{pc} = 3.086 \times 10^{16}\ \text{m}.$$

(1) $5.67 \times 10^{-8}\ \text{W m}^{-2}\,\text{K}^{-4} \to \text{mW cm}^{-2}\,\text{nK}^{-4}$

We perform the conversions step by step:

$$1\ \text{W} = 10^3\ \text{mW}, \quad 1\ \text{m}^2 = 10^4\ \text{cm}^2 \Rightarrow 1\ \text{m}^{-2} = 10^{-4}\ \text{cm}^{-2},$$

$$1\ \text{K} = 10^9\ \text{nK} \Rightarrow 1\ \text{K}^{-4} = 10^{-36}\ \text{nK}^{-4}.$$

Combining all factors:

$$5.67 \times 10^{-8}\ \text{W m}^{-2}\,\text{K}^{-4} \times 10^3 \times 10^{-4} \times 10^{-36} = \boxed{5.67 \times 10^{-45}\ \text{mW cm}^{-2}\,\text{nK}^{-4}}$$

(2) $1.055 \times 10^{-34}\ \text{J s} \to \mu\text{g km}^2\,\text{h}^{-1}$

Since $1\ \text{J} = 1\ \text{kg m}^2\,\text{s}^{-2}$, we have

$$1.055 \times 10^{-34}\ \text{J s} = 1.055 \times 10^{-34}\ \text{kg m}^2\,\text{s}^{-1}.$$

Now,

$$1\ \text{kg} = 10^9\ \mu\text{g}, \quad 1\ \text{m}^2 = 10^{-6}\ \text{km}^2, \quad 1\ \text{s}^{-1} = 3600\ \text{h}^{-1}.$$

Thus,

$$1.055 \times 10^{-34}\ \text{kg m}^2\,\text{s}^{-1} \times 10^9 \times 10^{-6} \times 3600 = \boxed{3.8 \times 10^{-28}\ \mu\text{g km}^2\,\text{h}^{-1}}$$

(3) $70\ \text{km s}^{-1}\,\text{Mpc}^{-1} \to \text{year}^{-1}$

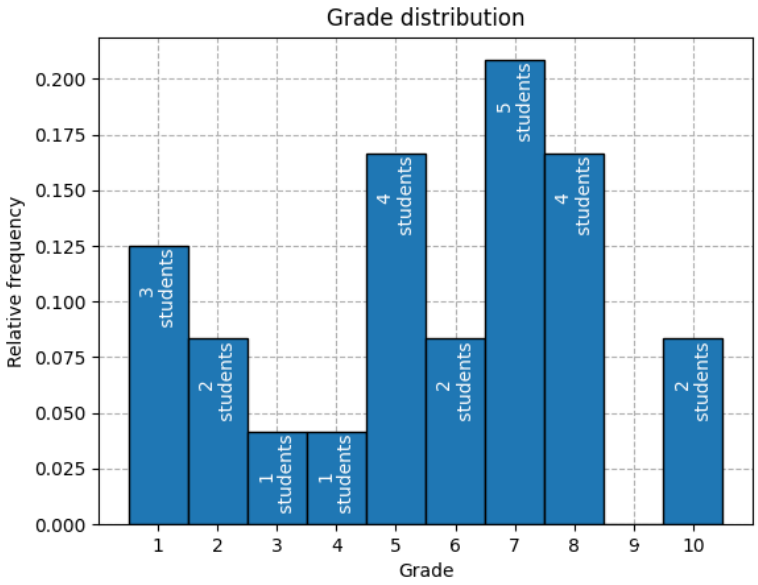
Using $1\ \text{Mpc} = 10^6\ \text{pc} = 3.086 \times 10^{22}\ \text{m}$, we get

$$70\ \text{km s}^{-1}\,\text{Mpc}^{-1} = 70 \times 10^3\ \text{m s}^{-1} \times (3.086 \times 10^{22}\ \text{m})^{-1} = 2.27 \times 10^{-18}\ \text{s}^{-1}.$$

Since $1\ \text{year} = 3.156 \times 10^7\ \text{s}$, we find

$$2.27 \times 10^{-18}\ \text{s}^{-1} \times 3.156 \times 10^7\ \text{s/year} = \boxed{7.2 \times 10^{-11}\ \text{year}^{-1}}$$

11. **Grade Distribution**
Knowing that 24 students have taken a test, estimate the number of them who have scored a 4 or lower, using the figure below.



Solution: Knowing, that the number of students is the frequency multiplied by the total number of students (i.e. 24):

We see that the fraction of students with a 1 is 0.125, which corresponds to $24 \cdot 0.125 = 3$ students.

For grade 2 the frequency is ≈ 0.08 which gives us $24 \cdot 0.08 = 1.92 \approx 2$ students.

The frequency for grades 3 and 4 is the same, ≈ 0.04, giving 2 times $24 \cdot 0.04 = 0.96 \approx 1$ student each.

Therefore, a total of $3 + 2 + 2 \cdot 1 =$ **7 students** have obtained a grade of 4 or lower.