Practice session: Interstellar Medium

Kapteyn Learning Community Sirius A
Give motivations and/or derivations for your answers.

1. **Warming-up questions**
   (a) Examples of heating mechanisms in the ISM are heating through photon ionization ($A+h\nu \rightarrow A^+ + e^-$) and heating via the photoelectric effect (ejection of electrons from dust grains). Explain two other heating mechanisms as well as two cooling mechanisms in the ISM.
   (b) What is a photo dissociation region (PDR)?
   (c) Give three examples of evidence for the existence of dust in the ISM.
   (d) Consider the formation of CH^+ in molecular clouds through these three channels:

$$C^+ + H \rightarrow CH^+ + h\nu, \qquad k_1 = 4.46 \times 10^{-17}\ T_2^{-0.5} \exp(-0.229 T_2^{-2/3})\ cm^3\ s^{-1} \quad (1)$$

$$CH + h\nu \rightarrow \begin{cases} CH^+ + e^- & (47\ \%) \\ C + H & (53\ \%) \end{cases}, \quad k_2 = 1.62 \times 10^{-9}\ s^{-1} \quad (2)$$

$$C^+ + H_2 + 0.40\ eV \rightarrow CH^+ + H, \quad k_3 = 1.5 \times 10^{-10} \exp(-4640/T)\ cm^3\ s^{-1} \quad (3)$$

   Name the reaction type of each of these reactions.

Solution:

(a) Heating mechanisms to choose from:

1. Dust gas heating (heating by H_2 formation on dust)
2. Cosmic-ray heating
3. X-ray heating
4. Turbulence
5. Ambipolar diffusion
6. Gravitational heating
7. Shock heating

Shopping list for cooling mechanisms:

1. Collisional de-excitation (dominant)
2. Recombination
3. Free-free emission

(b) Photodissociation region: The region between the HII cloud and the dense molecular cloud. In between, in order, we have the WNM and CNM.

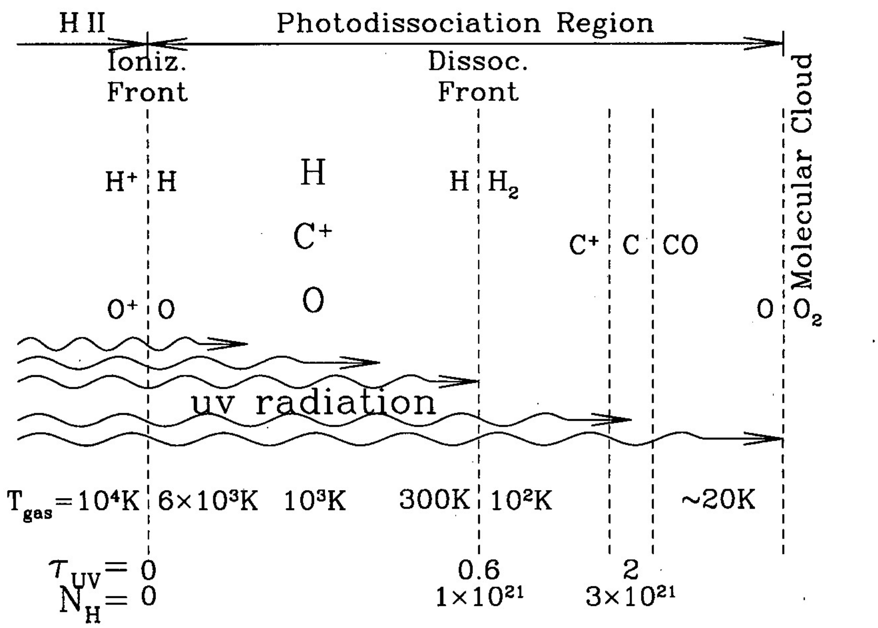


Figure 31.2 Structure of a PDR at the interface between an H II region and a dense molecular cloud.

(c) Here are four to choose from

- Direct imaging shows dark clouds which are so opaque which causes the blocking of background visible light. This is interpreted as observation by dust.
- Cool dust can emit infrared radiation and we can see this directly from IR satellite images.
- Looking at observations made, there is the conclusion that there is a higher H_2 abundance than we would expect. Therefore there has to be an other source of H_2, dust grains.
- Comparing luminosity distances and angular diameter distances of a sample of open clusters led to two conclusions. Angular diameter distances are systematically smaller than luminosity distances and distant clusters are redder. This are effects due to dust extinction.

(d) The three reaction mechanisms are:
(1) Radiative association
(2) Photoionization/dissociation
(3) Ion-neutral reaction

2. **Hydrogen clouds embedded in cold and warm neutral media**

We observe an interstellar cloud with a temperature $T = 100$ K, $n_H = 10^2$ cm^{-3}, and $n_e =$

10^{-1} cm^{-3}. An important emission line is the CII fine structure line at 157.74 μm. This line is produced because the ground state of ionized carbon splits into two fine structure levels $^2\text{P}^0_{1/2}$ and $^2\text{P}^0_{3/2}$. The Einstein A coefficient of the line transition between the two levels: $A_{10} = 2.4 \cdot 10^{-6}$ s^{-1}. The de-excitation rate due to collisions with electrons is $k_{10} = 4.53 \cdot 10^{-8} T_4^{-1/2}$ cm^3 s^{-1}. Where the temperature T_4 has units of 10^4 K.

(a) Explain the concept of critical density and how it relates to the condition of local thermodynamic equilibrium (LTE).

(b) Calculate the critical density, n_{crit}, of the 157.74 μm line.

(c) Calculate the collisional excitation coefficient, k_{01}, due to electrons using detailed balance for the hydrogen cloud.

(d) Calculate the relative populations, $\frac{n_1}{n_0}$, of the upper and lower level of carbon in the hydrogen cloud. Assume the two levels to be in statistical equilibrium and that the main collision partners are hydrogen atoms (with $k_{10} = 1.38 \cdot 10^{-9}$ cm^3 s^{-1}) and that the collision rate coefficients are in detailed balance.

(e) Now imagine finding this cloud in the cold neutral medium where typical turbulent velocities are 0.8 km s^{-1}. Calculate the full-width-half-maximum (FWHM) of the CII line in km s^{-1}.

(f) Calculate the FWHM of the HI 21 cm line in km s^{-1} assuming thermal and large scale turbulent motion broadening for the line profile, where v_{turb} follows a Maxwell-Boltzmann distribution.

(g) Explain with sketches the Voigt profile in the context of the previous two questions.

(h) Now instead imagine the hydrogen cloud in a warm neutral medium. First explain the two phases of the neutral medium. How does the pressure and temperature of the surroundings of the hydrogen cloud change when moving it from the cold to the warm neutral medium?

Solution:

(a) Density at which de-excitation equals excitation, so they are in local thermodynamic equilibrium. Often a sound approximation for optically thin lines, ignoring stimulated emission.

(b) The critical density can be calculated as:

$$n_{\text{crit}} = \frac{A_{10}}{k_{10}} = 52.98 T_4^{1/2} \text{ cm}^{-3} = 52.98 \cdot 0.1 = 5.298 \text{ cm}^{-3}. \tag{4}$$

(c) The other coefficient can be calculated by using:

$$k_{01} = \frac{g_1}{g_0} e^{-\frac{E_{10}}{kT}} k_{10}. \tag{5}$$

We know that the energy of the transition is $E = h\frac{c}{\lambda} \approx 1.259 \times 10^{-14}$ erg. If we look at the two states then we see that the degeneracies are:

$$g_1 = 2 \cdot \frac{3}{2} + 1 = 4, \tag{6}$$

$$g_0 = 2 \cdot \frac{1}{2} + 1 = 2. \tag{7}$$

Plugging those in we find:

$$k_{01} = 3.64 \cdot 10^{-7} \text{ cm} \tag{8}$$

(d) We know from the Boltzmann equation that:

$$\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{-\frac{E_{10}}{kT}} = 2e^{-\frac{1.26\times10^{-14}}{1.38\times10^{-16}\cdot 100}} = 0.8 \tag{9}$$

So, given the assumptions, for each 4 carbon atoms that are neutral there are on average five singly-ionized carbon states.

(e) The standard deviation of the velocity is:

$$\Delta V_C = \sqrt{\frac{kT}{m_C} + V_{\text{turb}}^2} = \sqrt{\frac{\cdot 1.38 \times 10^{-16} \cdot 100}{1.67 \cdot 10^{-24} \cdot 12} \cdot 10^{-10} + 0.8^2} = 0.84 \text{ km s}^{-1}. \tag{10}$$

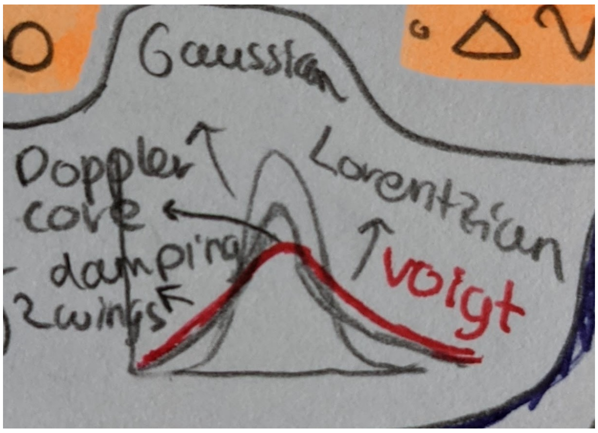
$$\text{FWHM} = 2\sqrt{2\ln 2}\Delta V_C = 1.98 \text{ km s}^{-1}. \tag{11}$$

(f) In the case of the hydrogen line we have:

$$\Delta V_{HI} = \sqrt{\frac{kT}{m_H} + V_{\text{turb}}^2} = \sqrt{\frac{\cdot 1.38 \times 10^{-16} \cdot 100}{1.67 \cdot 10^{-24}} \cdot 10^{-10} + 0.8^2} = 1.21 \text{ km s}^{-1}. \tag{12}$$

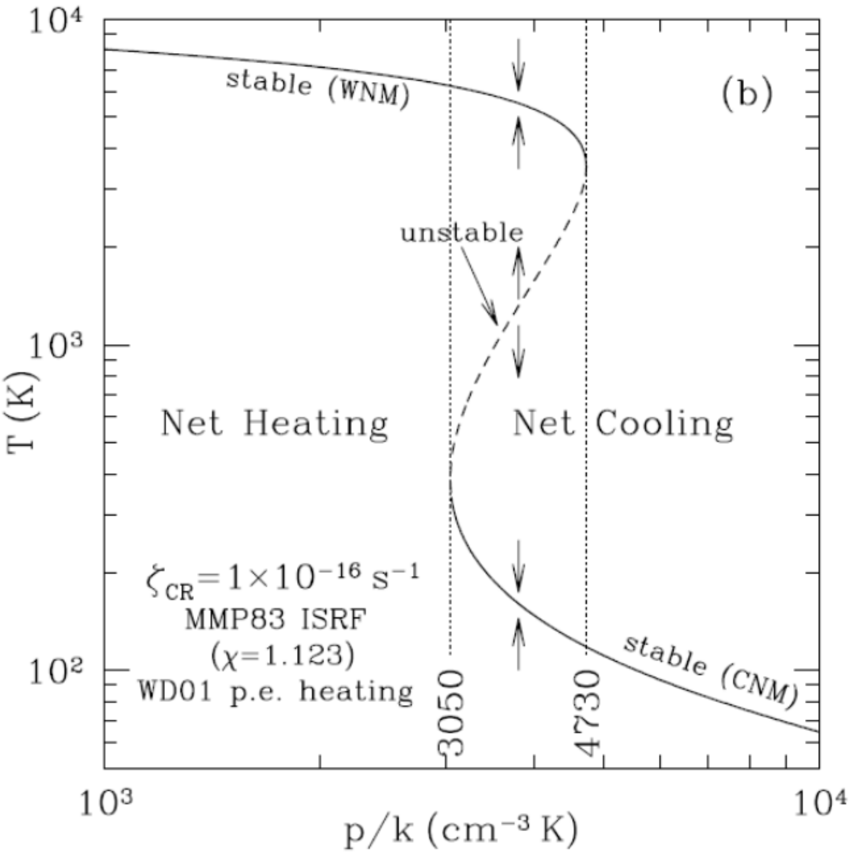
$$\text{FWHM} = 2\sqrt{2\ln 2}\Delta V_{HI} = 2.85 \text{ km s}^{-1}. \tag{13}$$

(g) The profile can be seen as a probability distribution which results from three broadening mechanisms. Firstly, one producing a Gaussian profile (due to **thermal** or **Doppler broadening**). Secondly, **Collisional broadening** between atoms causes gaps in the line profile. Finally, Lorentzian profile due to quantum effects. Also called **natural broadening**. The Voight profile is used to determine the intensity of an atomic absorption line. This means that this profile is the result of the previous two questions together with the quantum mechanical behavior of the line itself. The core of the profile is dominated by Doppler effects and the wings by the uncertainty principle and collisions.



Credits: H.C. Woudenberg

(h) There are two phases in the neutral IGM, see the figure below.



3. **Stellar winds, and ISM, and stuff...**

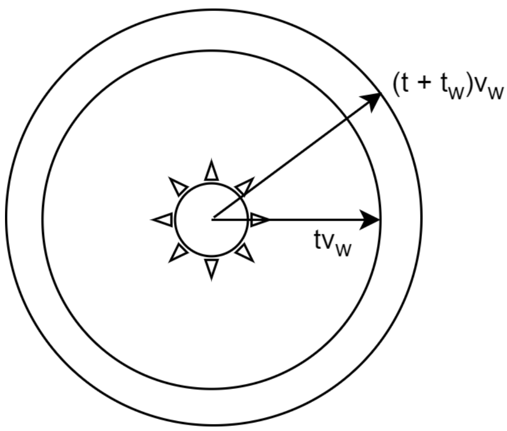
Consider a spherically symmetric stellar wind with mass-loss rate $\dot{M}_w = 10^{-4}$ M$_\odot$ yr^{-1}, and wind speed $v_w = 20$ km s^{-1}. Suppose that the mass loss continues steadily for a time $t_w = 10^3$ yr. After this time the wind no longer loses mass and it continues to move outwards. Suppose that after a time t, the central star suddenly becomes an ionizing source emitting hydrogen-ionizing photons at a rate Q_0. So, after a time $t + t_w$ the star starts to ionize the stellar wind. A protoplanetary nebula is created.

(a) After a time t, the outflowing wind has a spherical outer surface and a spherical inner 'hole'. What is the number density of particles just inside the outer surface?
Hint: What are the main constituents of this number density?

(b) What is the number density just outside the inner hole?

(c) Ignoring the expansion of the nebula during the ionization process, what is the minimum value of the ionizing photon rate, Q_0, required to ionize the hydrogen throughout the nebula? Given is the hydrogen recombination rate $\alpha = 2.59 \times 10^{-13}$ cm^3 s^{-1}.

(d) Line ratios can be used to estimate the electron density or temperature in HII regions. Explain which conditions the selection of the line system must fulfill in order to be suited for measuring



the electron temperature in the ionized nebula.

(e) If the HII region contains sufficient dust, photoelectric heating by dust could take place, explain this process.

(f) What is the recombination time just inside the outer surface? Compare this to the 10^3 year long dynamical age of the wind, t_w.

(g) What is the difference between the cases A and B recombination? Which of them is a good description of the recombination process in the ionized nebula?

Solution:

(a) We know the following things:

- Mass loss: $\dot{M}_w = 10^{-4}$ M$_\odot$ yr^{-1}.
- Wind speed: $V_w = 20$ km s^{-1}.
- Wind time: $t_w = 10^3$ yr.

After a time t the central star starts to emits hydrogen-ionizing photons at a rate of Q_0. We know the density is given by:

$$\rho = \frac{\mathrm{d}M}{\mathrm{d}V} = \frac{\frac{\mathrm{d}M}{\mathrm{d}t}\mathrm{d}t}{4\pi r^2 \mathrm{d}r} = \frac{\dot{M}_w}{4\pi r^2 V_w}. \tag{14}$$

But we actually want the number density, which can be written as follows:

$$n = \frac{\rho}{\mu m_H} = \frac{\dot{M}_w}{4\pi r^2 V_w \mu m_H}. \tag{15}$$

The mean molecular mass is given by:

$$\mu m_H = \frac{n_H m_H + n_{\mathrm{He}} m_{\mathrm{He}}}{n_H + n_{\mathrm{He}}} = \frac{14}{11} m_H = 1.27 m_H. \tag{16}$$

This means we can calculate the particle density just inside the outer shell:

$$
\begin{aligned}
n &= \frac{10^{-4}\ \mathrm{M}_\odot\ \mathrm{yr}^{-1}}{4\pi(t+t_w)^2 V_w^2 \cdot V_w \mu m_H}, && (17)\\
&= \frac{10^{-4}\cdot 1.989\cdot 10^{33}\ \mathrm{g\ yr^{-1}\ s}}{4\pi\left(\frac{t+t_w}{\mathrm{yr}}\right)^2 \mathrm{yr}^2 (20)^3\ \mathrm{km}^3\ \mathrm{s}^{-2}\cdot 1.27\cdot 1.67\cdot 10^{-24}\ \mathrm{g}}, && (18)\\
&= \frac{1.989\cdot 10^{29}\ \mathrm{g}\cdot \frac{1}{\pi\cdot 10^7}}{4\pi\left(\frac{t+t_w}{\mathrm{yr}}\right)^2 (20)^3\cdot(10^5\ \mathrm{cm})^3\cdot 1.27\cdot 1.67\cdot 10^{-24}\cdot(\pi\cdot 10^7)^2}, && (19)\\
&= \frac{1.989}{4\pi^4\cdot 8\cdot 1.27\cdot 1.67}\cdot\frac{10^{29}}{10^3\cdot 10^{15}\cdot 10^{-24}\cdot 10^{14}}\left(\frac{t+t_w}{\mathrm{yr}}\right)^{-2}, && (20)\\
&= 3\cdot 10^{17}\left(\frac{t+t_w}{\mathrm{yr}}\right)^{-2}\ \mathrm{cm}^{-3} && (21)
\end{aligned}
$$

(b) The second calculation goes similarly with the only difference being $t_w = 0$. Then the particle number density equals:

$$n = 3\cdot 10^{17}\left(\frac{t}{\mathrm{yr}}\right)^{-2}\ \mathrm{cm}^{-3} \qquad (22)$$

(c) In this part we consider only the photons with an energy higher than 13.6 eV. Use the following relation for the ionization rate:

$$Q_0 = \int n(H^+) n_{eH} \alpha \mathrm{d}V. \qquad (23)$$

Further, we use $n(H^+) = n_e$, $n(H^+) = \frac{10}{11}\frac{\rho(r)}{\mu}$. This means we can calculate the minimum

Q_0 as:

$$Q_0 = \int_{tV_w}^{(t+t_w)V_w} n_H^2 \alpha 4\pi r^2 \mathrm{d}r, \tag{24}$$

$$= \int_{tV_w}^{(t+t_w)V_w} \left(\frac{10}{11} \frac{\dot{M}_w}{4\pi r^2 V_w \mu m_H} \right)^2 \alpha 4\pi r^2 \mathrm{d}r, \tag{25}$$

$$= \left(\frac{10}{11}\right)^2 \frac{\dot{M}_w^2 \alpha}{4\pi V_w^2 \mu^2 m_H^2} \left(-\frac{1}{r}\right)\Big|_{tV_w}^{(t+t_w)V_w}, \tag{26}$$

$$= 0.7 \left(\frac{(10^{-4}\ \mathrm{M_\odot\ yr^{-1}})^2 \cdot (2.59 \cdot 10^{-13}\ \mathrm{cm^3\ s^{-1}})}{4\pi (20 \cdot 10^5\ \mathrm{cm\ s^{-1}})^3 \left(\frac{1.4}{1.1}\right)^2 (1.67 \cdot 10^{-24}\ \mathrm{g})^2} \right) \left(-\frac{1}{t+t_w} + \frac{1}{t} \right), \tag{27}$$

$$= 5 \cdot 10^{50}\ \mathrm{s^{-1}} \left(\frac{1}{\frac{t-t_w}{\mathrm{yr}}} - \frac{1}{\frac{t}{\mathrm{yr}}} \right), \tag{28}$$

$$= 5 \cdot 10^{50}\ \mathrm{s^{-1}} \left(\frac{t-(t-t_w)}{t^2 - t_w t} \right), \tag{29}$$

$$= 5 \cdot 10^{50}\ \mathrm{s^{-1}} \left(\frac{t_w}{\frac{t^2 - t_w t}{\mathrm{yr}}} \right). \tag{30}$$

(d) $n_{\mathrm{crit}} \ll n_e$, and upper energy levels need to be separated by $\sim kT$.

(e) Dust grains absorb FUV photons, which can lead to the ejection of an electron from the grain. The electron carries excess kinetic energy which will be transferred to the gas as thermal energy. The smallest dust grains dominate this method of heating.

(f) The recombination time scale is defined as

$$\tau_{\mathrm{rec}} = \frac{1}{\alpha_B n_H} \tag{31}$$

Where $n_H = \frac{10}{11} \frac{\rho}{\mu m_H}$. If we calculate this we obtain:

$$\tau_{\mathrm{rec}} = \frac{1}{\alpha_B \left(\frac{10}{11} \frac{11}{14} \frac{\rho}{m_H} \right)}, \tag{32}$$

$$= \left(\frac{t+t_w}{\mathrm{yr}} \right)^2 \cdot 4.6 \cdot 10^{-6}\ \mathrm{yr}. \tag{33}$$

This is much shorter than the dynamical time.

(g) Case A is an ideal situation which assumes that all lines are optically thin, while case B doesn't take into account the Lyman series because they will be re-absorbed immediately because the cloud is optically thick for these lines. Since the recombination time is much smaller than the dynamical time, case B fits best.

4. **A problem that does not change much in time**

Consider the following radiative recombination reaction of hydrogen

$$\mathrm{H} + \mathrm{e}^- \rightarrow \mathrm{H}^- + \mathrm{h}\nu, \quad \mathrm{k}_r = 1.9 \cdot 10^{-16}\ \mathrm{T}_2^{0.67}\ \mathrm{cm}^3\ \mathrm{s}^{-1} \tag{34}$$

where k_r is the rate coefficient. Furthermore, consider the reaction

$$\mathrm{H}^- + \mathrm{H} \rightarrow \mathrm{H}_2 + \mathrm{e}^-, \quad \mathrm{k}_a = 1.3 \cdot 10^{-9}\ \mathrm{cm}^3\ \mathrm{s}^{-1} \tag{35}$$

which is a fast ion-neutral reaction with rate coefficient k_a. Note that H^- undergoes photoionization as well in the interstellar radiation field:

$$\mathrm{H}^- + \mathrm{h}\nu \rightarrow \mathrm{H} + \mathrm{e}^-, \quad \xi_{pd} = 2.4 \cdot 10^{-7}\ \mathrm{s}^{-1} \tag{36}$$

with the rate ξ_{pd}. Consider a neutral hydrogen cloud with density $n_H = 30\ \mathrm{cm}^{-3}$ and electron density $n_e = 0.02\ \mathrm{cm}^{-3}$.

(a) What is the steady-state ratio $\frac{n(H^-)}{n_H}$?

(b) What fraction of the H^- ions undergo the ion neutral reaction?

(c) Evaluate the quantity:

$$R_{H^-} \equiv \frac{k_a \cdot n(H^-) \cdot n(H)}{n_H \cdot n(H)} \tag{37}$$

Compare this to the 'effective rate coefficient' for formation of H_2 by dust grain catalysis.

(d) Consider that the HI cloud would start to collapse and cool down. The cloud contains a high fraction of dust, what would be a good column density tracer?

(e) Explain what the X-factor is and what it could be used for in this cloud.

Solution:

(a) To determine the steady state we have:

$$\frac{\mathrm{d}n(H^-)}{\mathrm{d}t} = n(H^0)n(e^-)k_{ra} - n(H^-)n(H^0)k_{ad} - n(H^-)\xi_{pd} = 0 \tag{38}$$

Which implies by steady state that:

$$n(H^0)n(e^-)k_{ra} - n(H^-)\left(n(H^0)k_{ad} - \xi_{pd}\right) = 0. \tag{39}$$

In the case of a HI cloud we can simply say that: $n_H = n(H^0)$, this implies that:

$$\frac{n(H^-)}{n_H} = \frac{n(e^-)k_{ra}}{n_H k_{ad} + \xi_{pd}} \tag{40}$$

(b) The fraction can be calculated as:

$$X = \frac{k_{ad} n(H^0) n(H^-)}{k_{ad} n(H^0) n(H^-) + \xi_{pd} n(H^-)}, \tag{41}$$

$$= \frac{k_{ad} n(H^0)}{k_{ad} n(H^0) + \xi_{pd}} \tag{42}$$

again we used the assumption $n_H = n(H^0)$. Further calculations result in:

$$X = \frac{k_{ad} n_H}{k_{ad} n_H + \xi_{pd}}, \tag{43}$$

$$= \frac{1.3 \cdot 10^{-9}\ \text{cm}^3\ \text{s}^{-1} \cdot 30\ \text{cm}^{-3}}{1.3 \cdot 10^{-9}\ \text{cm}^3\ \text{s}^{-1} \cdot 30\ \text{cm}^{-3} \cdot 2.4 \cdot 10^{-7}\ \text{s}^{-1}} = 0.14. \tag{44}$$

(c) From (a) we know $n(H^-)/n_H$, therefore we get:

$$R_{H^-} = k_{ad} \cdot 1.36 \cdot 10^{-11} T_2^{0.67}, \tag{45}$$

$$= 1.8 \cdot 10^{-20} T_2^{0.67}\ \text{cm}^3\ \text{s}^{-1}. \tag{46}$$

Comparing the value to the 'effective rate coefficient' for formation of H_2 by dust grain catalysis $R(H_2) = 3 \cdot 10^{-17}$ cm^3 s^{-1}. We see that the formation of H is relatively slow. Formation through this mechanism is inefficient, the formation of H_2 by dust grain catalysis is much faster and more efficient.

(d) CO would be a good density tracer.

(e) The X-factor is an empirical determined relation between the CO density and the H_2 density, this means that by using CO we could determine the H_2 density.