Advanced Electrodynamics, Homework Set 3

Deadline: Thursday February 29, 12:00 (noon)

1. a. Give the fields $\vec{E}(x,y,z,t)$ and $\vec{B}(x,y,z,t)$ of an electromagnetic plane wave traveling in the x-direction, polarized in the y-direction, of angular frequency ω, and with amplitude E_0.

   b. The same wave is observed in an inertial frame moving in the x-direction with speed v relative to the frame of part a. Find the electric and magnetic fields $\vec{E}'(x',y',z',t')$ and $\vec{B}'(x',y',z',t')$.

   c. Calculate the frequency, wavelength, and speed of the wave in the frame of b.

   d. Calculate the ratio of the intensities of the wave in the two frames. What happens when $v \to c$?

2. a. Consider the electromagnetic wave of problem 1a. Show that its fields can be obtained from a suitable four-potential $A^\mu = (V/c, \vec{A}) = (0, 0, A_y, 0)$.

   b. Consider this wave observed in a frame moving in the y-direction with speed v relative to the frame of part a. Evaluate again the frequency and wavelength of the wave in this frame. Calculate the fields in this frame. Verify that the wave is linearly polarized and that the fields in this frame can be obtained from a four-potential $A^{\mu\prime} = (0, \vec{A}')$.

   c. Find the four-potential $\tilde{A}^{\mu\prime}$ obtained from A^μ with a Lorentz transformation. Verify that the fields of part b. can also be obtained from $\tilde{A}^{\mu\prime}$. Show that $A^{\mu\prime}$ and $\tilde{A}^{\mu\prime}$ are related by a gauge transformation.

3. a. Prove the continuity equation directly from the Maxwell equation $\partial_\nu F^{\mu\nu} = \mu_0 J^\mu$.

   b. Show that the equation $\partial_\nu G^{\mu\nu} = 0$ is equivalent to $\partial_\lambda F_{\mu\nu} + \partial_\mu F_{\nu\lambda} + \partial_\nu F_{\lambda\mu} = 0$.

   a. Express the Lorentz scalars $F_{\mu\nu}F^{\mu\nu}$, $F_{mu\nu}G^{\mu\nu}$, and $G_{\mu\nu}G^{\mu\nu}$ in terms of $\vec{E}$ and $\vec{B}$.

   b. Can a purely electric field in one inertial frame appear as a purely magnetic field in some other frame?

4. In the Compton effect, an electron at rest is hit by a photon, which changes the momentum of both. Find the energy of the emitted photon in terms of its incident energy and the angle of ideviation from its initial path. Use four-vectors.

5. A positron can be made by colliding an electron at rest with a photon: $\gamma + e^- \to e^- + e^+ + e^-$. What is the minimum photon energy required? Use four-vectors and invariant combinations of them where possible. Is it possible to make an electron-positron pair with the reaction $\gamma \to e^+ + e^-$?