Advanced Electrodynamics, Homework Set 2

Deadline: Thursday February 22, 12:00 (noon)

1. Express the "permittivity" and the "permeability" of the vacuum, $\epsilon_0 \simeq 8.85 \cdot 10^{-12}$ F/m and $\mu_0 \simeq 1.26 \cdot 10^{-6}$ H/m, respectively, in base SI units and show that $c \simeq 3 \cdot 10^8$ m/s.

2. A layer of radioactive material is deposited on the surface of a sphere. The material emits charged α particles of high energy. Suppose that the particles are emitted radially outward from the surface of the sphere. Does this current produce a magnetic field?

3. Show explicitly that the retarded solutions for the potentials V and $\vec{A}$ satisfy the Lorenz condition.

4. Suppose that the current density $\vec{J}$ is constant in time.

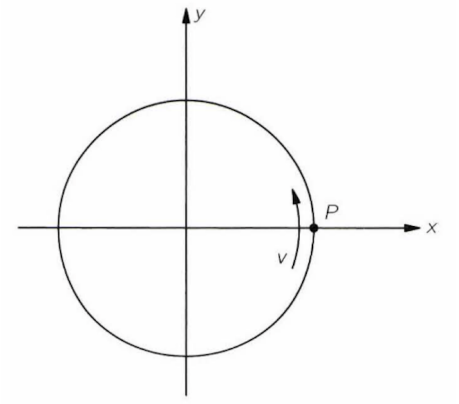
   a. Use the continuity equation to show that the charge density ρ is a linear function of time.
   b. Show that the electric field $\vec{E}$ obeys Coulomb's law with ρ evaluated at the non-retarded time.
   c. Show that the magnetic field $\vec{B}$ obeys Ampère's law with Maxwell's displacement current added.

5. Suppose that the current density $\vec{J}$ changes slowly enough in time that we can write

   $$\vec{J}(t_r) = \vec{J}(t) + (t_r - t)\dot{\vec{J}}(t) \ ,$$

   where t_r is the retarded time. Show that the magnetic field $\vec{B}$ obeys the Biot-Savart law, with $\vec{J}$ evaluated at the non-retarded time.

6. A particle with charge q moves in a circle of radius R with constant, relativistic speed v. We want to calculate the electric and magnetic fields when the particle is at point P (see the Figure).



   a. Find the scalar and the vector potential, V and $\vec{A}$, at the centre of the circle.
   b. From the potentials, find $\vec{E}$ and $\vec{B}$ at the centre of the circle.